

Discussion of:

**"A Stochastic Discount Factor Approach to  
Asset Pricing Using Panel Data Asymptotics"**

by

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## Summary

- ▶ New econometric nonparametric estimator of the SDF based on panel-data asymptotic: Araujo Issler (AI) SDF.
- ▶ Contribute to a large literature started by Hansen and Jagannathan (HJ, 1991), where nonparametric SDFs are used to diagnose models, test predictability, performance analysis, ...
  - ▶ On each state, the AI SDF is proportional to the inverse of the geometric mean of cross sectional returns.
  - ▶ Main advantage with respect to HJ and its variations is the simplicity of estimation for a large number of assets. However, ...
  - ▶ it comes with a cost of introducing large pricing errors, **but not much larger than the errors achieved by previously proposed SDFs.**
- ▶ Empirical application: Tests of parametric SDFs by projections on their proposed SDF.

## Comparisons with the Literature

- ▶ The AI nonparametric SDF should be compared to a number of other previously proposed nonparametric/semiparametric SDFs:
  - ▶ 1) Nonparametric Hansen and Jagannathan (1991) linear SDF with positivity constraint, and all its generalizations with conditional variables
  - ▶ 2) Nonlinear SDF by Bansal and Viswanathan (1993) obtained with Logistic Functions
  - ▶ 3) Taylor expanded SDF by Dittmar (2002)
  - ▶ 4) Stutzer (1995) exponential, Growth portfolio SDF by Bansal and Lehman (1997)
- ▶ Most of these SDFs present small/zero pricing errors and are also based on economic theory.

## The AI SDF is too smooth: It is not Admissible

- ▶ The AI SDF presents high pricing errors, and I believe this is the reason for why the Equity Premium Puzzle disappears!
- ▶ An important diagnostic test: See if the SDF is above the HJ variance bound with positivity constraint.
  - ▶ As I show below, this is not the case. The sample volatility of the AI SDF (0.0215) is 16 times smaller than the volatility of the HJ SDF with pos. constraint (0.3588)!
- ▶ The AI SDF is not useful to test parametric SDFs like the CCAPM by projections since it already does not explain a significant amount of information on returns.
- ▶ But it is still a good approximation for an admissible SDF when the number of assets is large.

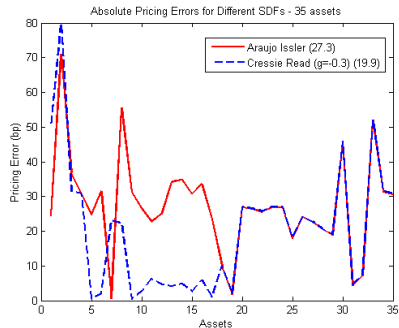
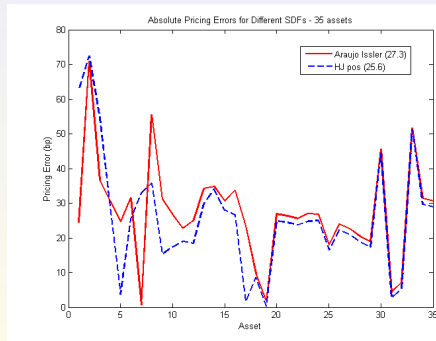
## Robustness of Results: Comparison with HJ with positivity constraint

- ▶ HJ propose a linear SDF with positivity constraint that is simple to estimate, if the number of assets is not too large
  - ▶ This SDF also introduce pricing errors when the number of priced assets increase.
  - ▶ The HJ estimator becomes harder to solve when the number of priced assets increase.
- ▶ Therefore, an effective comparison is fundamental to understand the advantages and drawbacks of the new methodology.
  - ▶ Extracting the risk-free rate, analyzing the pricing errors from Euler equations, pricing out-of-sample in the panel, etc...
- ▶ I compare the AI SDF with the HJ SDF with positivity constraint and with some other SDFs.

# Euler Equation Pricing Errors

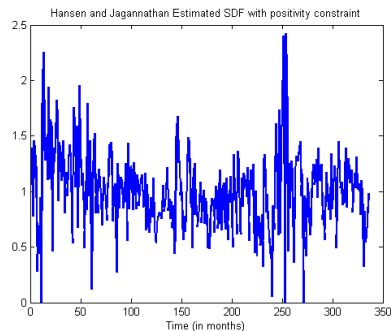
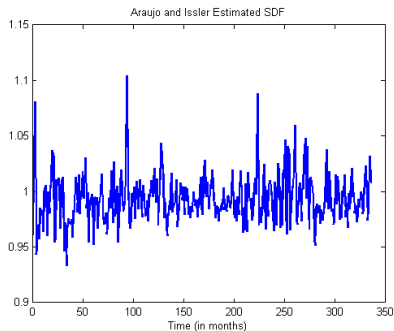
- SDFs should satisfy the Euler Equation for the returns  $R$ :

$$E[mR] = 1$$



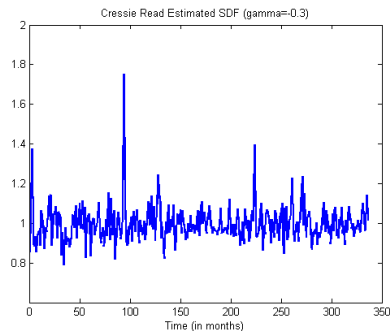
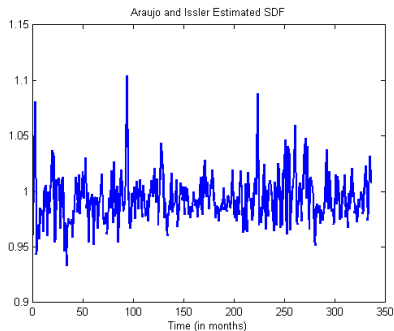
# Comparing SDF's Shape

- ▶ The HJ SDF is much more volatile than the AISDF:  $0.3588 \times 0.0215$
- ▶ The absolute average pricing errors are a bit smaller:  $25.6 \text{ bp} \times 27.3 \text{ bp}$



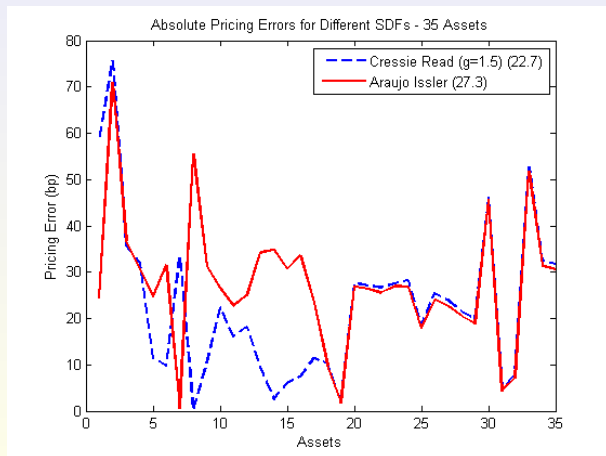
# Comparing SDF's Shape

- ▶ The CR ( $\gamma = -0.3$ ) SDF is more volatile than the AISDF: 0.0877 x 0.0215
- ▶ The absolute average pricing errors are much smaller: 19.9 bp x 27.3 bp



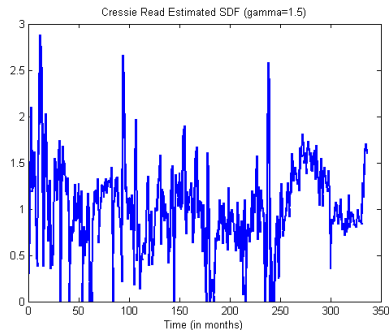
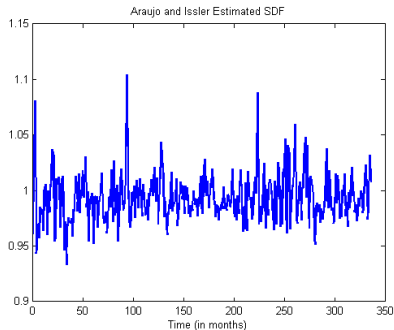
# More on Euler Equation Pricing Errors

- ▶ SDFs should satisfy the Euler Equation for the returns  $R$ :  
 $E[mR] = 1$



## Comparing SDF's Shape: Much more volatility...

- ▶ The CR ( $\gamma = 1.5$ ) SDF is much more volatile than the AISDF:  $0.4774 \times 0.0215$
- ▶ The absolute average pricing errors are smaller: 22.7 bp x 27.3 bp



# Obtaining An Infinity of Positive Semi-parametric SDFs Based on Nonlinear Projections

- ▶ Almeida and Garcia (2010) solve SDF nonlinear projection problems where the discrepancy functions (distances) are based on members of the Cressie Read family.
- ▶ Implied SDFs are obtained from the first order conditions of portfolio problems based on HARA utility functions.
- ▶ The implied SDFs are strictly positive hyperbolic functions of the primitive returns, and admissible, that is, perfectly price the primitive assets:

$$\hat{m}_{MD}(R) = \beta * \left( 1 + \gamma \hat{\lambda}'_{opt} \left( R - \frac{1}{a} \right) \right)^{\frac{1}{\gamma}} \quad (1)$$

- ▶ Include Stutzer (1995) exponential, Growth portfolio SDF by Bansal and Lehman (1997), and HJ (1991) as particular cases

## Conclusion

- ▶ AI SDF is simple to estimate and presents pricing errors comparable to other well known nonparametric SDFs.
- ▶ In particular, it has volatility too small, much smaller than the minimum volatility required for an SDF to be admissible.
- ▶ On the other hand, most estimated SDFs, in a context of a large number of assets, are much harder to obtain and also present large pricing errors.
- ▶ AI SDF appears to be a particularly promising candidate to give approximately right prices when there is a large set of assets.