

Discussion of:

**“Affine Term Structure Models, Volatility and
the Segmentation Hypothesis”
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Summary of the Paper

- Investigate the ability of affine term structure models to capture interest rates volatility.
- Different specifications of three-factor affine models are studied adopting U.S. Treasury zero coupon yields.
- Empirical results indicate that conditional volatilities implied by affine models are positively correlated with EGARCH volatilities for all maturities:
 - **Higher correlations for long-term maturities (around 80%) and lower for short-term maturities (around 60%).**
 - **Indication of a segmentation of volatility on the maturity spectrum (different time-scales).**
- Contrast to results provided on previous studies (CDGJ (2006), and AB (2005)).

1. What to expect from successful dynamic term structure models

1. Conditional Variance and (Covariances) of interest rates for different maturities varies through time.

- Dai and Singleton (RFS, 2003) suggest (1) as a challenge.
- Collin Dufresne et al. (WP, 2006) capture varying conditional variance with a four factor affine model.
- Bester (WP, 2004) captures time varying covariance with an infinite dimensional term structure model.

2. Expected excess returns are time-varying: Excess ret. are predictable.

- Duffee (JF, 2002) and Dai & Singleton (JFE, 2002) show that multi-factor Gaussian models can account for this fact when pricing swap rates.
- Almeida et al. (2006) obtain that the A1(3) estimated with options (caps) is the best candidate to predict long-term swaps excess returns.

3. They should price cross section of bonds and options.

- Jagannathan et al. (JE, 2003) show that a 3-factor CIR performs poorly when attempting to price caps.
- Bikbov and Chernov (WP, 2004) compare different affine models estimated with Eurodollars, and Eurodollar options.
- Han (JF,2006) prices swaptions and approximately captures swaps prices, with an specific USV model.

2. Modeling Volatility of interest Rates with DTSMs

- There are some papers that deal with this task:
- Directly, considering USV:
 - Andersen and Benzoni (2005), CDGJ (2006), Joslin (2006), and Thompson (2004).
- Directly, not considering USV:
 - Bester (2004).
- Indirectly
 - Bikbov and Chernov (2004), Han (2006), and Almeida et al. (2006)
- 6 out of 8 consider USV models.
- Wouldn't it be interesting to consider USV models on your comparisons?
 - The cost would be to adopt an alternative methodology to estimate the USV models like in CDGJ (2006) or Thompson(2004), but...
 - The gain would be a direct comparison to some of the previous papers and possibly even better results when matching volatilities.

3. Are three Factors Enough to Model Volatility?

- The answer appears to be negative according to the results within this paper:
 - Short-term volatility is not completely captured.
 - There appears to be a necessity of two sources of uncertainty modeling volatility (the so called segmentation effect proposed in the paper)
- These results are completely in line with the results appearing in Joslin (2006).
- First he shows that any incompleteness in affine models should come from USV.

- Then provides theoretical arguments indicating that three and four factor models with USV (with only one volatility factor) are not able to capture conditional volatilities of both short- and long-term yields.
- Combining all the previous information we obtain that:
- **If there is any incompleteness one must adopt a four-factor USV model!**
- Might be interesting to compare USV and non-USV four factor models.

4. The Importance of the Market Price of Risk When Modeling Volatility

- Flexible Market Prices of Risk (MPR) is important for reconciling time series and cross sectional properties, specially when time series properties are related to first moment restrictions (Dai and Singleton (2002), Duffee (2002), Duarte (2004)).
- However, it shouldn't be specifically important to capture historical volatility (see CDGJ (2006) pag. 3).
- Unless option data is adopted, in which case, volatility risk premia might be identified through more general specifications of MPR (see Almeida et al. (2006)).

- My intuition is partially confirmed with results from Tables 1, 2 and 3 where the majority of MPR parameters appear to be statistically insignificant:
- - **No significance for all completely affine models**
 - **2** significant out of **6** param. for the A_1(3) E.A model.
 - **None** significant out of **3** param. for the A_2(3) E.A.
 - **1** significant param. out of **7** for the A_1(3) E.E.A.
 - **2** significant out of **8** param. for the A_3(3) E.E.A.
- If MPRs were important on this quest they would probably have higher significance.

5. Reconciling the Apparently Opposite Findings...

- A point appearing recurrently in the article is that CDGJ (2006) and Andersen & Benzoni (2005) reach diametrically opposite conclusions to yours.
- CDGJ (2006) focuses only on the volatility of the short-rate and their result is similar to the segmentation hypothesis result that you propose:
 - There is an incompleteness coming from something similar to USV on short-term maturities.
- You surely have different quantitative results (different correlations signs) but the idea of segmentation (two volatility factors) with USV appears to be common to CDGJ (2006), Jacobs and Karoui (2006), and Joslin (2006).

6. Related research projects worth looking at

1. Bester (2004), who compares affine models to random fields in terms of ability to capture volatility and correlations of interest rates, and find that affine models do a poor job, specially with correlations.
2. Han (2006) proposes a string model very similar in spirit to affine models, with USV, and with completely affine market prices of risk. His model simultaneously captures caps and swaptions prices.

7. Possible Directions of Research...

1. Include USV models and/or possibly four-factor models on the empirical investigation.

2. A Combination of rich MPR specifications with the “time series” econometric methodology proposed by Thompson (2004)...
 - could be useful to verify if affine models can be successful in matching simultaneously the failure of EH and time varying conditional volatilities.

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