Interest Rate Risk Measurement in Brazilian Sovereign Markets

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Abstract

Fixed income emerging markets are an interesting investment alternative. Measuring market risks is mandatory in order to avoid unexpected huge losses. The most used market risk measure is the Value at Risk, based on the profit-loss probability distribution of the portfolio under consideration. Estimating this probability distribution requires the prior estimation of the probability distribution of term structures of interest rates. An interesting possibility is to estimate term structures using a decomposition of the spread function into a linear combination of Legendre polynomials. Numerical examples from the Brazilian sovereign fixed income international market illustrate the practical use of the methodology.

Key words: Emerging Markets, Interest Rate, Risk Management.

JEL Classification: C51, C52, F21, G15.

Resumo

Os mercados emergentes de renda fixa são alternativas interessantes para investimento. A medição dos riscos de Mercado é necessária para que se evite um nível elevado de perdas. A medida de risco de mercado mais utilizada é o *Value at Risk,* baseado na distribuição de perdas-ganhos da carteira sob análise. A estimação desta distribuição requer, no entanto, a estimação prévia da distribuição das estruturas a termo das taxas de juros. Uma possibilidade interessante para a estimação da distribuição das estruturas a termo das taxas de juros é efetuar uma decomposição da função de *spread* em uma combinação linear de Polinômios de Legendre. Exempos numéricos do mercado internacional de títulos soberanos brasileiros são apresentados para ilustrar o uso prático desta nova metodologia.

Palavras-Chave: Mercados Emergentes, Taxas de Juros, Gerenciamento de Riscos.

1. Introduction

Fixed income emerging markets developed quickly during the last decade. Higher international liquidity, the interest of portfolio managers in diversifying internationally, and the continuous improvement of risk control by international rating agencies are three reasons for such a development.

Some of the most liquid instruments traded in fixed income emerging markets are the so-called Brady bonds (Fabozzi and Franco (1997)). They are dollar denominated sovereign instruments, originated from the restructuring of defaulted bank loans of countries located in Latin America, Central and Eastern Europe, Middle East, Africa and Asia.

Pricing and hedging these instruments is not easy due to their usually complex cash flows. They may present floating (for instance, depending on the LIBOR rates) or step up interest payments, amortize or capitalize principal before maturity, contain embedded options, as well as offer collateralized principal and/or interest payments.

Other fixed income instruments in the emerging debt market include bank loans, local issues, and eurobonds. Eurobonds are bonds issued in a foreign currency, in a foreign country. Interest on them has been growing steadily due to the improving credit rating of certain emerging markets, with special attention directed, more recently, to Global bonds, which are eurobonds issued simultaneously in several countries.

Whenever investing in emerging markets one must pay special attention to risk management. Controlling risk is mandatory in order to avoid unexpected high losses. The Value at Risk (VaR; Jorion (2001)) of a portfolio, obtained as a percentile of the profit-loss probability distribution of the portfolio, is one of the most frequently adopted market risk measures. On its turn, the profit-loss probability distribution is closely related to the distribution of term structures of interest rates probability distribution. For instance, if we are interested in calculating the VaR of fixed income portfolios, we need first to estimate the probability distribution of term structures of interest rates.

Vasicek and Fong (1982) suggests estimating the U.S term structure of interest rates applying a regression model based on exponential splines. Litterman and Scheinkman (1991) verifies, using Principal Component Analysis (PCA; Mardia *et al.* (1992)), that more than 90% of the U.S term structure of interest rates movements were explained by just three orthogonal factors. Several applications were proposed in the finance literature following that. For instance, Singh (1997) uses PCA to estimate the market risk of fixed income instruments in the US market. Barber and Copper (1996) proposes an immunization strategy also based in PCA to generate optimal hedges.

Almeida *et al.* (1998) suggests a modeling approach for term structures of interest rates in emerging markets. The model is based on a decomposition of the term structure in a risk free benchmark curve plus a spread function representing the sovereign credit risk spread. This spread function, on its turn, is decomposed into a linear combination of Legendre polynomials (Lebedev (1972)). An extension to this model, which considers the relative credit risk among the assets included in the estimation process, is possible (Almeida *et al.* (2000)). In this extension, the relative credit risk is captured by considering the rating of each asset in the evaluation of the credit spread function. This enhanced methodology provides more accurate estimates of term structures, at the expense of more computational complexity.

In this article, a methodology for estimating the VaR of portfolios in fixed income emerging markets is proposed. We estimate the historical probability distribution for the variations of the benchmark curve, as well as for each orthogonal factor responsible for movements of the spread function, obtaining the historical probability distribution of variations for the whole term structure. Two numerical examples using data from the Brazilian sovereign fixed income market are presented. These examples illustrate the VaR estimation of a portfolio composed by Brazilian Brady and Global bonds.

The article is organized as follows. Section 2 presents the model for the estimation of term structures of interest rates in emerging markets, and the methodology used in our examples. Section 3 presents the numerical examples. Section 4 concludes the article.

2. Term Structure of Interest Rates in Emerging Markets

2.1. Definition

The term structure of interest rates in a fixed income emerging market can be modeled as:

$$R(t) = B(t) + \sum_{n \geq 0} c_n P_n \left(\frac{2t}{\ell} - 1\right), \ \forall t \in [0, \ell]. \qquad (1)$$

where t denotes time, $B(t)$ is a benchmark curve (for instance, the U.S. term structure of interest rates), P_n is the Legendre polynomial of degree n (Lebedev (1972)), c_n is a parameter, and ℓ is the longest maturity of a bond in the emerging market under consideration.

The price of a bond (p) relates to the term structure by:

$$p = \sum_{i=1}^{n_A} C_i \exp(-t_i R\ (t_i))$$

(2)

where C_i denotes the cash flow paid by the bond on time t_i , and n_A denotes the total number of cash flows paid by the bond.

The Legendre polynomial of degree n is defined (in the compact set $[-1,1]$) according to the following expression:

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}((x^2 - 1)^n)\ , \ \forall \ \ n = 0,1,2... \ . \qquad (3)$$

The first four Legendre polynomials are:

$$
\begin{aligned}
P_0(x) &= 1 \\
P_1(x) &= x \\
P_2(x) &= \frac{1}{2}(3x^2 - 1) \\
P_3(x) &= \frac{1}{2}(5x^3 - 3x).
\end{aligned} \tag{4}
$$

Their graphs are depicted in Figure 1. The first Legendre polynomial will be related to parallel shifts in the term structure, the second Legendre polynomial will be related to changes in slope of the term structure, the third Legendre polynomial will be related to changes in curvature of the term structure, and the fourth Legendre polynomial will be related to double changes in curvature of the term structure.

[Figure 1 about here]

2.2. Estimation

The first step required is to estimate the coefficients $c_0, c_1, c_2, \ldots,$ in Equation (1).

We define the discount function as:

$$D(t) = e^{-R(t)t}, \forall t \in [0, \ell]. \tag{5}$$

We assume that m bonds are available in a particular instant of time for the estimation process. The estimation of the coefficients $c_0, c_1, c_2, \ldots,$ is accomplished by the use of a non-linear regression equation given by:

$$p_j + a_j + 1_j^{put} o^p - 1_j^{call} o^c = \sum_{l=1}^{f_j} u_{jl} D(t_{jl}) + e_j, \forall j = 1,2,\ldots,m, \tag{6}$$

where p_j denotes the price of the j^{th} bond, a_j denotes the accrued interest of the j^{th} bond, 1_j^{put} and 1_j^{call} are dummy variables indicating the existence of embedded put and call options in the bond, o^p and o^c are unknown parameters related to the prices of the embedded put and call options, f_j denotes the number of remaining cash flows of the j^{th} bond, and t_{jl} the time remaining for payment of the l^{th} cash flow u_{jl} of the j^{th} bond; e_j is the random disturbance, with $E(e_j) = 0$, $E(e_j^2) = s_j^2$, $E(e_j, e_i) = 0 \, \forall j \neq i$.

Table 1 presents the characteristics of fourteen Brazilian bonds (Brady and Global) used in the numerical examples. Figure 2 depicts two term structures of interest rates estimated using the model just described: one term structure for Brady bonds, and the other term structure for Global bonds.

[Table 1 and Figure 2 about here]

2.3. Joint Estimation of Term Structures of Interest Rates of the Brady and Global Bonds Market

As an example of the methodology just described, let us suppose that we are interested in estimating the Brazilian Brady and Global bonds term structures of interest rates. These two types of assets may present, during certain periods of time, substantially different levels of credit risk. Thus, estimating a unique term structure of interest rates to represent both markets may "distort" results. On the other hand, estimating separately one curve for each market may present statistical difficulties in the cases where the number of liquid assets belonging to one of these markets is small. In order to avoid these drawbacks, we apply the methodology presented in Almeida *et al.* (2000), which captures the difference in risk between different levels of credit risk using different credit spread functions. The result is a joint estimation procedure that estimates simultaneously the two term structures.

Consider the existence of two levels of credit risk represented by r_G, r_B, where the first is related to the Global bonds market, and the second to the Brady bonds market. The methodology proposes an extension to Equation (1) to capture different levels of credit risk considering the spread function depending explicitly on these levels:

$$R(t, r_G, r_B) = B(t) + C(t, r_G, r_B), \forall t \in [0, \ell]. \quad (8)$$

The spread function is still modeled as a linear combination of Legendre polynomials. The model is completely specified when the dependence on different levels of credit risk is defined. For instance, it is possible to capture the difference in credit risk using just the translation factor (Legendre polynomial of degree zero). A consequence from such a specification is:

a) The term structures of interest rates estimated simultaneously using the model differ only by parallel shifts.

Or, equivalently:

b) When comparing all maturities of these term structures, the volatilities of the interest rates spreads differ only by a constant value.

For the first numerical example presented in this article we used a more general model. It allows the interest rates spreads of one term structure with respect to the other to present volatility differing not only by a constant value along the maturities. This flexibility can be achieved by allowing the term structures to possess also different curvatures. Thus, the model captures the difference in credit risk using the Legendre polynomials of degree zero and two (which are responsible, respectively, for parallel shifts and changes in curvature of the term structure of interest rates).

The equations for the term structures of interest rates for the Global and Brady markets are respectively (letting $\tilde{t} = \frac{2t}{\ell} - 1, \forall t \in [0, \ell]$):

$$R^G(t) = B(t) + \sum_{n \geq 0} c_n^G P_n(\tilde{t}). \quad (9)$$

$$R^B(t) = B(t) + \left(c_0^G + c_0^B\right) P_0(\tilde{t}) + c_1^G P_1(\tilde{t}) + \left(c_2^G + c_2^B\right) P_2(\tilde{t}) + \sum_{n \geq 3} c_n^G P_n(\tilde{t}). \quad (10)$$

The coefficients c_0^B and c_2^B allow the Brady term structure to present a distinct decomposition when compared to the Global term structure, with respect to parallel shifts and changes in curvature. In this particular case, the two curves present the same rotation factor with respect to the benchmark curve. Later in this article we shall investigate the use of different rotation factors.

Figure 2 depicts the Brazilian Brady and Global bonds term structures of interest rates estimated on November 10, 2000, based on the model just described, with the U.S strips playing the role of the benchmark curve. Values for the first three orthogonal factors for each curve, with their p-values, are given in Table 2.

[Table 2 about here]

3. Estimating the Value at Risk of Two Brazilian Fixed Income Portfolios

Suppose we wanted to estimate the Value at Risk of two portfolios on November 10, 2000, using the model just described.

Portfolio 1 presents the following composition (see also Table 1):

a) Long US$ 20 million in CBOND.

b) Long US$ 20 million in DCB.

c) Long US$ 10 million in GLB30.

d) Short US$ 20 million in EI.

e) Short US$ 15 million in IDU.

f) Short US$ 15 million in GLB01.

Portfolio 2 is composed by the same bonds and the same amounts as Portfolio 1, but it presents only long positions:

a) Long US$ 20 million in CBOND.

b) Long US$ 20 million in DCB.

c) Long US$ 10 million in GLB30.

d) Long US$ 20 million in EI.

e) Long US$ 15 million in IDU.

f) Long US$ 15 million in GLB01.

As mentioned before, we need first to estimate the probability distribution of the variations of the term structures. We apply the Historical Simulation approach (Jorion (2001)) for estimating the interest risk in these two portfolios.

Obtaining the historical joint probability densities of the variations of the U.S strips term structure, and of all orthogonal factors in Table 2, is a computer intensive step. It requires running an optimization procedure for each day in the database to estimate the values of the orthogonal factors and, then, to estimate the historical term structures for Brady Bonds and Global Bonds. After obtaining the distributions for the orthogonal factors and for the U.S Strips, we can obtain the Brady and Global bonds term structures scenarios required by the Historical Simulation approach. In the numerical examples two hundred and fifty historical scenarios were generated. For each scenario, the associated term structures were used to price all bonds in the portfolios. At the end, we obtained the historical probability density of bond prices.

Let $\{q_i\}_{i=1,\dots,m}$ denote the random variables which represent the returns of the prices of the bonds, $\{w_i^{(j)}\}_{i=1,\dots,m}$ denote the amounts (in US$) of each bond in the j^{th} proposed portfolio, and V_j denotes the random variable which measures the profits and losses (in US$) of the j^{th} portfolio, for $j = 1,2.$. We constructed the probability density of the profits and losses of the j^{th} portfolio by multiplying the returns of the bonds listed in this portfolio by the amounts held on them:

$$V_j = \sum_{i=1}^{m} w_i^{(j)} q_i \qquad (11)$$

Finally, based on the probability densities of the random variables $\{q_i\}_{i=1,...,m}$ and $\{V_j\}_{j=1,2}$ (obtained by historical simulation) we estimate the VaR of both the proposed portfolios and of each bond used in the estimation process.

For instance, Figure 3 depicts the marginal probability densities of the returns of the translation, rotation and torsion orthogonal factors related to the Brady and Global term structures of interest rates. Note from Table 3 that the five historical distributions in Figure 3 violate the hypothesis of normality. They all present kurtosis greater than three, as well as non-zero skewness.

[Table 3 and Figure 3 about here]

Figure 4 and Figure 5 present, respectively, the probability densities of the returns of Portfolio 1 and Portfolio 2. Note that both are skewed to the left and present fat left-tail, meaning that there is a greater probability of loosing extreme values than earning extreme values. For the reasons detailed in Duarte (1997), the Analytical Approach (Jorion (2001)) is not recommended to compute the VaR of these two portfolios.

[Figure 4 and 5 about here]

Table 4 presents the Value at Risk for each bond used in the estimation process, for two distinct confidence levels: 99% and 95%. Similarly, Table 5 presents the Value at Risk for the proposed portfolios for 99% and 95% confidence levels. Table 6 shows the price correlation matrix for the bonds used in the estimation process.

[Table 4, 5 and 6 about here]

For the sake of illustration, we present two other models to decompose the spread of the Brady over the Global term structure. Table 7 presents the combinations of factors for each analyzed model. Model 1 corresponds to the model where the difference in risk is captured using only the translation factor. Model 2 captures the difference in credit risk using the translation and rotation factors. Finally, Model 3 represents the model used so far in this article (which is included here only for comparative purposes).

We provide in Figure 6 and Figure 7 the historical evolution of Brazilian Brady and Global term structures during one year of analysis, for Model 1 and Model 2, respectively.

[Table 7, Figure 6 and 7 about here]

Observing these graphs, we are capable of capturing which are the most important factors responsible for interest rate risk. For instance, Figure 6 and Figure 7 reveals that the Brady rotation factor is more volatile in Model 2 than in the others. This fact is in accordance with the specification of the model, which uses an extra free variable related to the rotation factor to describe the spread of the Brady over the Global term structure. Observing now the Global term structures, Figure 6 reveals that the rotation and torsion factors become more important as risk factors for more recent observations of the time series for Model 1. On the other hand, Figure 7 indicates that the Global term structure suffers significant changes in its curvature since the beginning of the time series for Model 2. These pictures represent an interesting tool for identification of the regions where the scenarios for the evolution of the term structures might produce the most extreme movements.

Table 8 presents the estimated VaR for Portfolios 1 and 2 for the three models. It generalizes what was observed in Table 5 for Model 3: Portfolio 2 presents higher risk than Portfolio 1, for all models, for both the 99% and 95% confidence levels. Another interesting fact is related to the distribution of mass in the left tails of the density functions of the returns of Portfolios 1 and 2. Note that if a model generates the higher risk, among all models, for a fixed confidence level, it does not mean that this model generates the highest risk for another fixed confidence level. Observe also that the difference in estimated risk existent comparing the models can be very large. For instance, if we compare the VaR for Portfolio 2, at a 99%

confidence level, estimated by Models 1 and 3, we identify a difference of 30% (US$ -5,230,000 and US$ –3,680,000). These remarks indicate the importance of observing the risk for different confidence levels, and also using models presenting different sources of risk to better capture the magnitude of possible losses, avoiding model risk when measuring market risk (Duarte (1997)).

[Table 8 about here]

4. Conclusion

We propose a methodology for estimating the Value at Risk of portfolios in fixed income emerging markets. It exploits the dynamics of the orthogonal factors, obtained by the decomposition of the credit spread function into a linear combination of Legendre polynomials.

This methodology produces a probability density function for the term structures of interest rates. It is possible to show that the use of the model described in Section 2.4 (to estimate the historical evolution of the whole term structures of emerging markets) generates a dynamic equivalent to the one obtained by using Principal Component Analysis in a market presenting an observable term structure. In other words, the methodology proposes the application of Principal Component Analysis in markets presenting non-observable term structures, which is the case of fixed income emerging markets. This fact allows us to use this methodology for, at least, all fixed income applications that may apply Principal Component Analysis, which is the case of risk analysis, portfolio allocation, immunization techniques etc.

Although the portfolios presented in the numerical examples were composed by only Brazilian fixed income instruments, the methodology can be easily extended to other financial markets (such as the U.S. corporate bond market).

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Figure 1. Four Legendre Polynomials

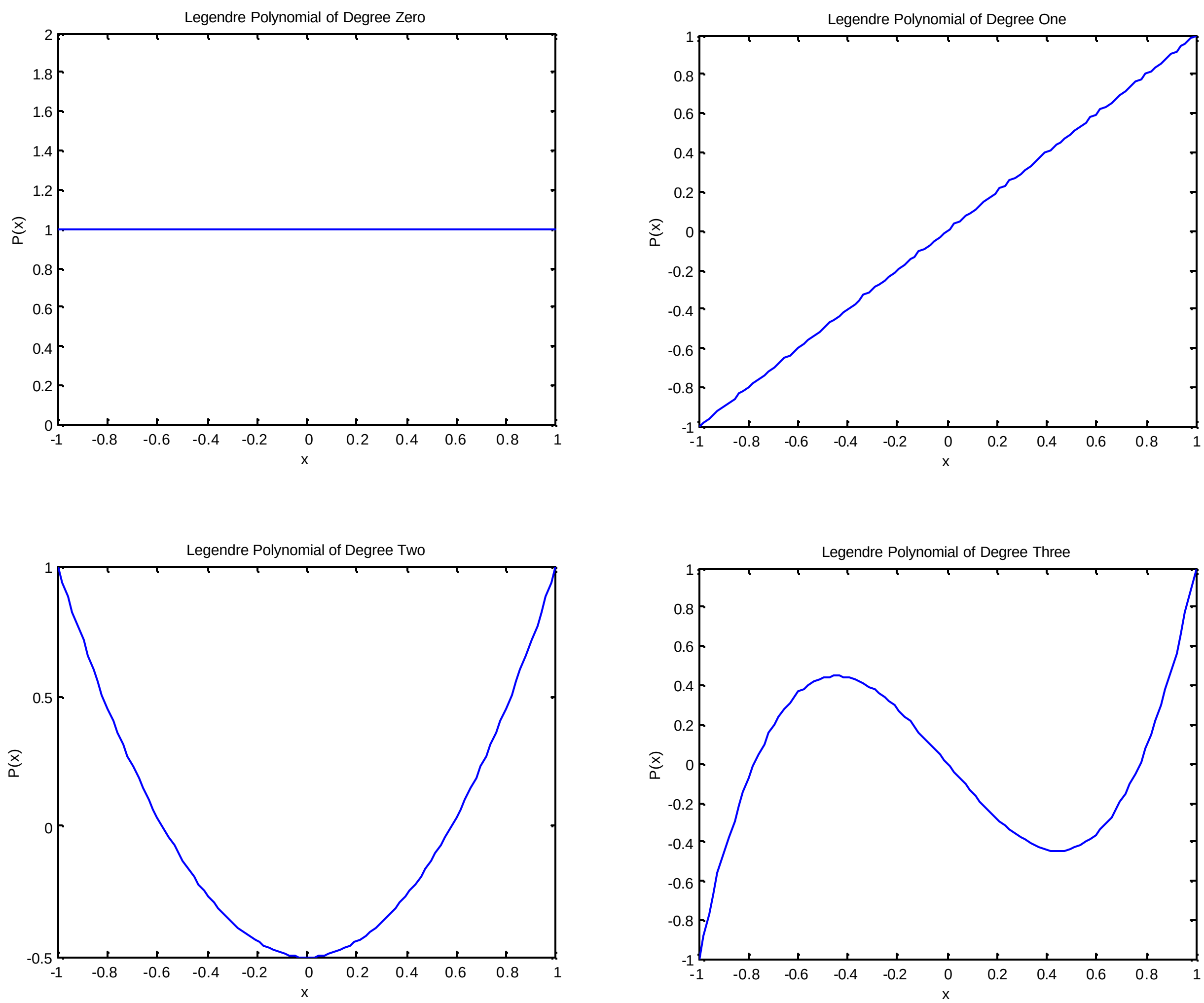

Table 1. Brazilian Bonds Used in the Estimation Process

Bond	Type	Coupon (%)	Duration (Years)	Maturity
CBOND	Brady	8.000	5.42	15-Apr-2014
DCB	Brady	7.440	5.13	15-Apr-2012
DFA	Brady	7.760	2.95	15-Sep-2007
EI	Brady	7.380	2.49	15-Apr-2006
IDU	Brady	7.840	0.13	01-Jan-2001
NMB	Brady	7.440	3.39	15-Apr-2009
GLB01	Global	8.875	0.95	05-Nov-2001
GLB04	Global	11.63	2.89	15-Apr-2004
GLB08	Global	9.380	5.25	07-Apr-2008
GLB09	Global	14.50	5.29	15-Oct-2009
GLB20	Global	12.75	6.59	15-Jan-2020
GLB27	Global	10.13	7.03	15-May-2027
GLB30	Global	12.25	7.01	06-Mar-2030
GLB40	Global	11.000	6.91	17-Aug-2040

Figure 2. A Joint Estimation of Term Structures of Brady and Global Markets

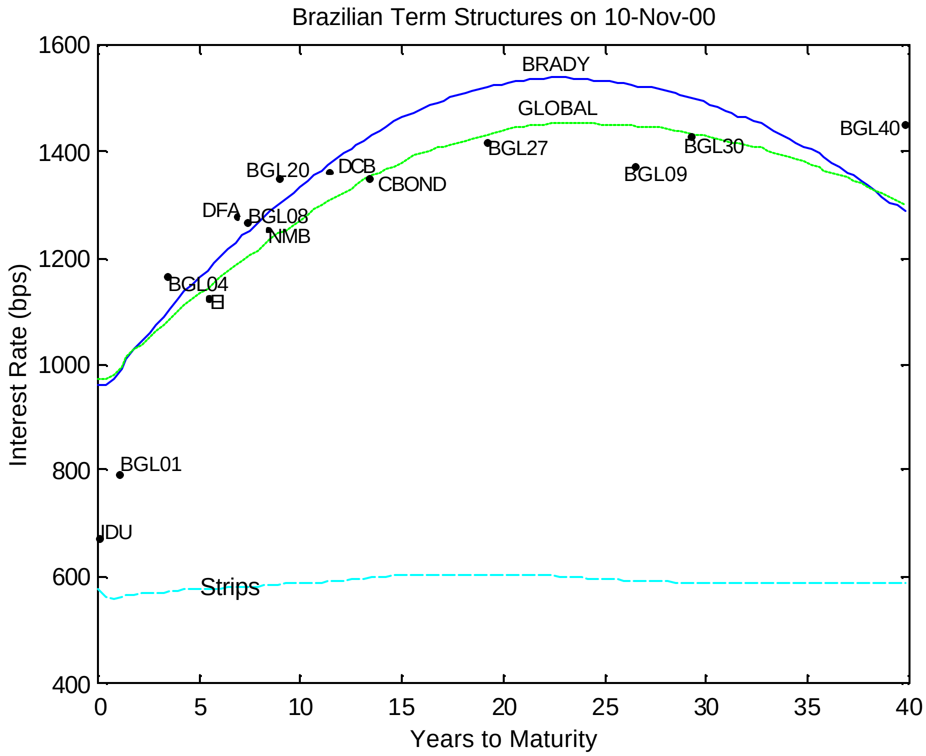

Table 2. Orthogonal Factors for Two Brazilian Term Structures

Factor[*]	Value (bps)	P-Value
Brady Translation	**747**	**0.000**[***]
Global Translation	**697**	**0.000**[***]
Rotation	**147**	**0.000**[***]
Brady Torsion	**-239**	**0.014**[**]
Global Torsion	**-174**	**0.000**[***]

[*]The first three Legendre polynomials correspond respectively to the translation, rotation and torsion factors.
[**]Statistically significant at a 5% confidence level.
[***]Statistically significant at a 1% confidence level.

Figure 3. Marginal Probability Density Functions of the Orthogonal Factors

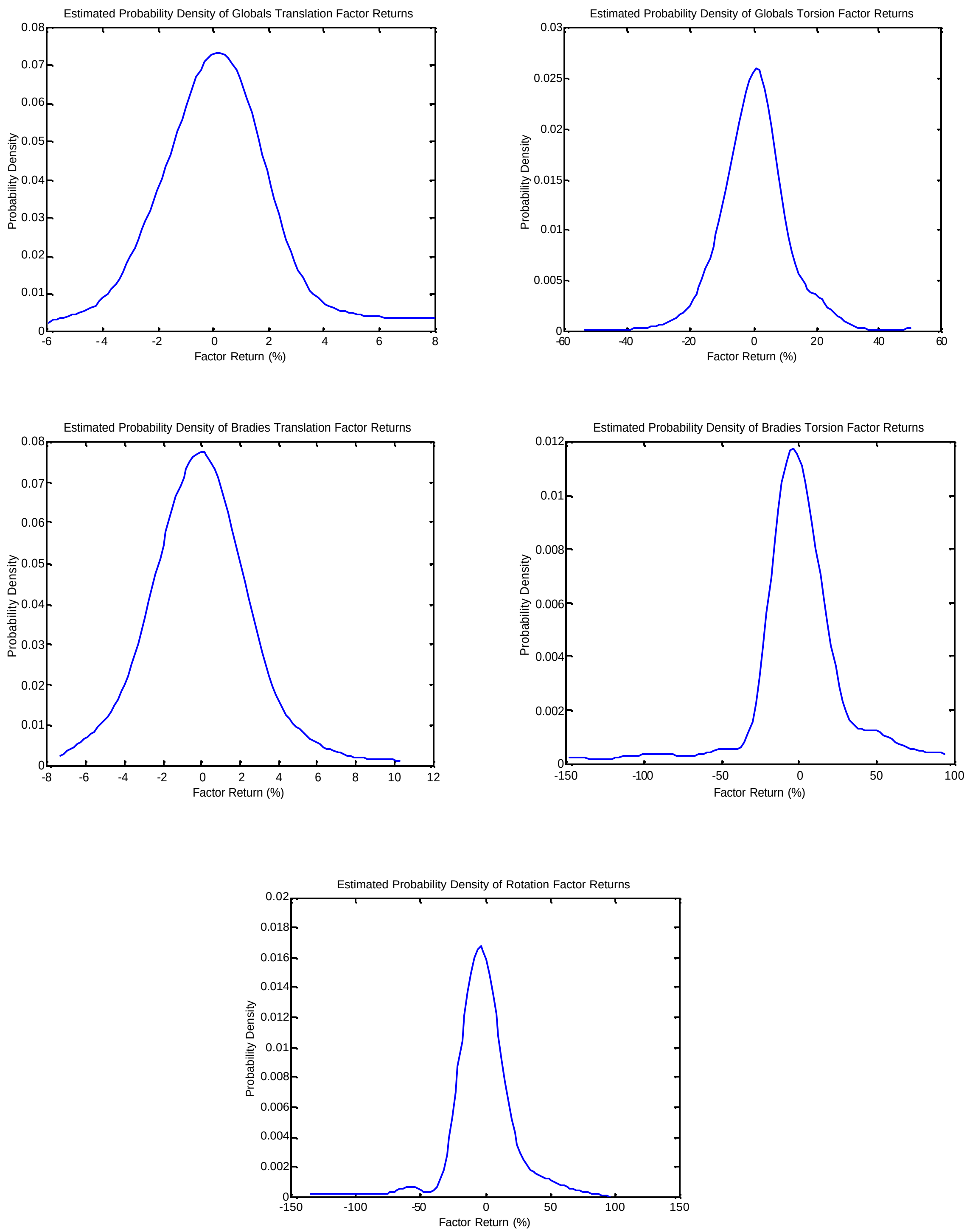

Table 3. Jarque Bera Normality Test for the Factors Returns

Probability Density of	Skewness	Kurtosis	p-value
Global Translation	0.11	3.95	0.006*
Global Torsion	-0.21	5.43	0.000*
Brady Translation	0.25	3.79	0.004*
Brady Torsion	-0.23	5.61	0.000*
Rotation	0.14	4.85	0.000*

*Reject the hypothesis of normality at a 1% significance level.

Figure 4. Estimated Probability Density Function of the Returns of

Portfolio 1

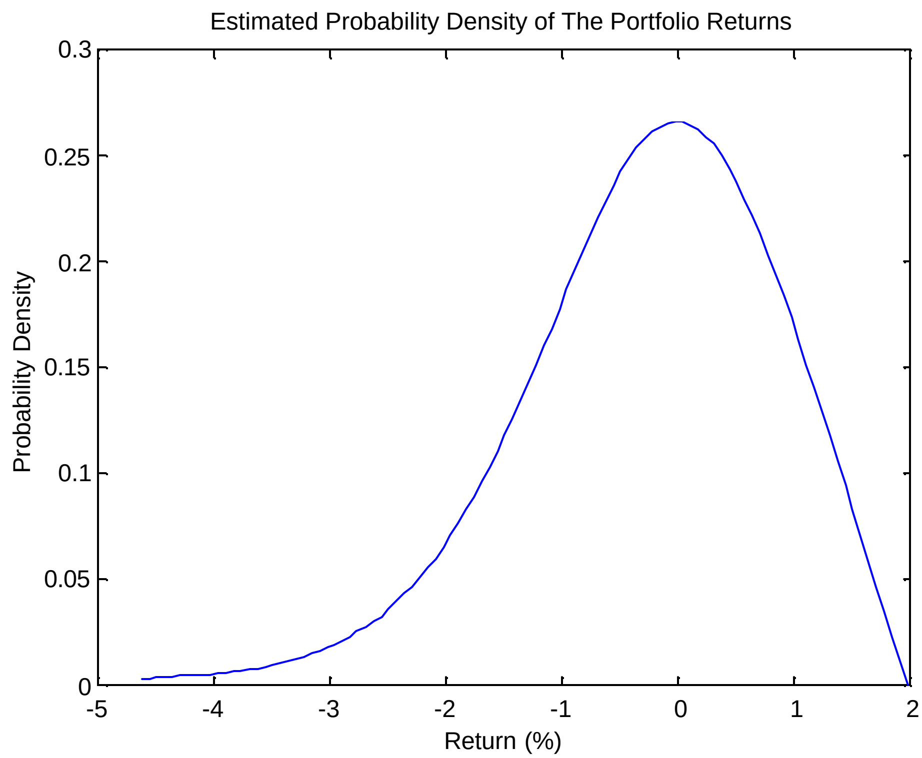

Figure 5. Estimated Probability Density Function of the Returns of

Portfolio 2

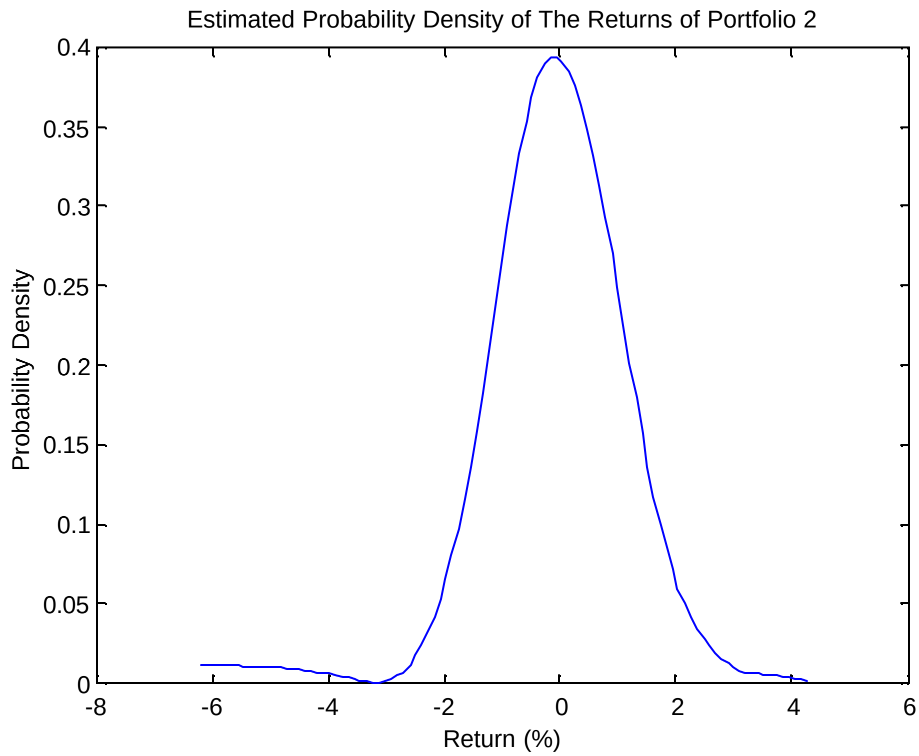

Table 4. Estimated Value at Risk for Global and Brady Bonds

Bond	Value at Risk (99%)	Value at Risk (95%)
CBOND	-6.05%	-3.02%
DCB	-6.16%	-3.17%
DFA	-5.78%	-3.47%
EI	-5.76%	-3.27%
IDU	-0.63%	-0.23%
NMB	-6.18%	-3.43%
GLB01	-2.91%	-0.60%
GLB04	-7.40%	-1.38%
GLB08	-8.21%	-2.24%
GLB09	-7.12%	-2.16%
GLB20	-6.50%	-2.29%
GLB27	-5.83%	-2.08%
GLB30	-5.92%	-2.14%
GLB40	-4.72%	-2.13%

Table 5. Portfolios Estimated Value at Risk Based on Historical Simulation

Portfolio	Value at Risk (99%)	Value at Risk (95%)
1	US$ -1,880,000	US$ -770,000
2	US$ -3,680,000	US$ -1,850,000

Table 6. Correlation Matrix for the Bonds Used in the Estimation Process

	CBOND	DCB	DFA	EI	IDU	NMB	GLB01	GLB04	GLB08	GLB09	GLB20	GLB27	GLB30	GLB40
CBOND	1.00	0.99	0.88	0.85	0.74	0.91	0.40	0.51	0.68	0.70	0.71	0.70	0.70	0.65
DCB	0.99	1.00	0.93	0.90	0.82	0.96	0.37	0.47	0.63	0.65	0.65	0.64	0.64	0.59
DFA	0.88	0.93	1.00	1.00	0.97	1.00	0.24	0.31	0.41	0.42	0.42	0.40	0.40	0.35
EI	0.85	0.90	1.00	1.00	0.98	0.99	0.21	0.27	0.36	0.37	0.36	0.35	0.35	0.30
IDU	0.74	0.82	0.97	0.98	1.00	0.95	0.12	0.17	0.23	0.23	0.23	0.21	0.21	0.17
NMB	0.91	0.96	1.00	0.99	0.95	1.00	0.26	0.34	0.46	0.47	0.47	0.45	0.45	0.40
GLB01	0.40	0.37	0.24	0.21	0.12	0.26	1.00	0.98	0.79	0.74	0.45	0.43	0.45	0.56
GLB04	0.51	0.47	0.31	0.27	0.17	0.34	0.98	1.00	0.90	0.86	0.62	0.60	0.61	0.70
GLB08	0.68	0.63	0.41	0.36	0.23	0.46	0.79	0.90	1.00	1.00	0.89	0.88	0.88	0.89
GLB09	0.70	0.65	0.42	0.37	0.23	0.47	0.74	0.86	1.00	1.00	0.93	0.91	0.92	0.91
GLB20	0.71	0.65	0.42	0.36	0.23	0.47	0.45	0.62	0.89	0.93	1.00	1.00	1.00	0.94
GLB27	0.70	0.64	0.40	0.35	0.21	0.45	0.43	0.60	0.88	0.91	1.00	1.00	1.00	0.96
GLB30	0.70	0.64	0.40	0.35	0.21	0.45	0.45	0.61	0.88	0.92	1.00	1.00	1.00	0.96
GLB40	0.65	0.59	0.35	0.30	0.17	0.40	0.56	0.70	0.89	0.91	0.94	0.96	0.96	1.00

Table 7. Estimating VaR Using Different Combinations of Factors

Model	Translation	Rotation	Torsion
1	X		
2	X	X	
3	X		X

Figure 6. Historical Brazilian Term Structures for Model 1

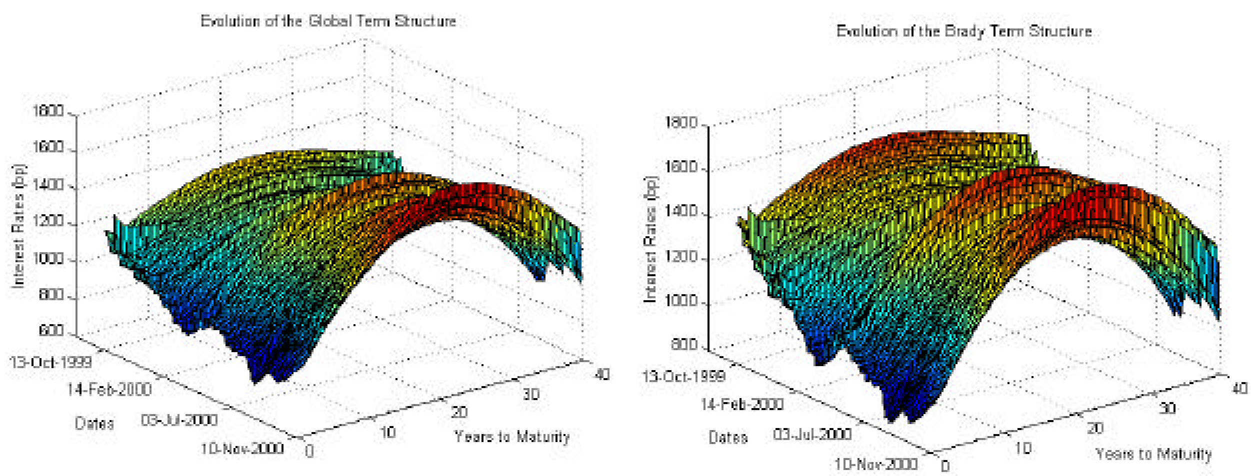

Figure 7. Historical Brazilian Term Structures for Model 2

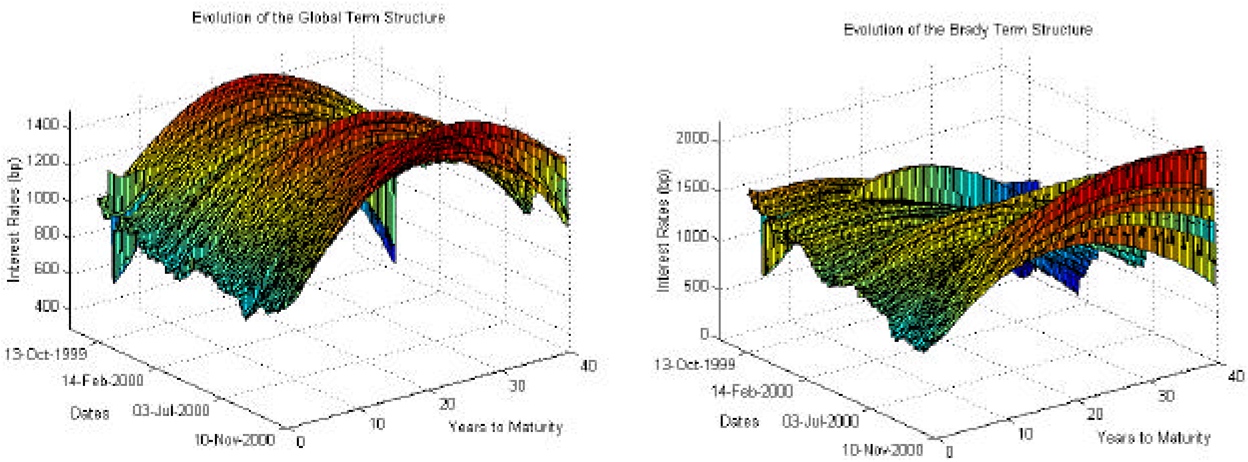

Table 8. Estimated Value at Risk Based on Historical Simulation

Portfolio 1

Model	Value at Risk (99%)	Value at Risk (95%)
1	US$ -1,800,000	US$ -880,000
2	US$ -2,680,000	US$ -1,870,000
3	US$ -1,880,000	US$ -770,000

Portfolio 2

Model	Value at Risk (99%)	Value at Risk (95%)
1	US$ -5,230,000	US$ -1,760,000
2	US$ -4,960,000	US$ -2,600,000
3	US$ -3,680,000	US$ -1,850,000