Early Childhood Education and Economic Growth*

Bruno Ricardo Delalibera†

Pedro Cavalcanti Ferreira‡

November 6, 2018

Abstract

We study the effects of early childhood skill formation on productivity and schooling. We add early childhood human capital to a standard continuous time life cycle economy and assume complementarity between educational stages. Agents are homogenous and choose the intensity of preschool education, how long to stay in formal school, labor effort and consumption. The model is calibrated to the U.S. from 1961 to 2008 and matches the data very well and closely reproduces the paths of schooling, hours worked, relative prices and GDP. We find that early childhood education can explain a large part of the observed increase of years of schooling in the U.S. since 1961, and it was as important as formal education for the increase of labor productivity in the period. Furthermore, we show that small reallocations of public expenditures from formal education to early childhood education would have sizable impacts on income per capita and productivity.

Keywords: Preschool education; schooling; growth; productivity.

JEL: O11; O40; I24.

*We thank the editor, B. Ravikumar, an anonymous referee, Diego Restuccia, Cezar Santos, Samuel Pessôa, Marcelo Santos, Ricardo Cavalcanti, Fernando A. de Barros Jr., Valdemar de Pinho Neto, Luiz Brotherhood, Heron Rios and seminar participants of the SBE 2016, 2rd International REAP and SBE Meetings and the EPGE macroeconomic group for very helpful suggestions. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. We also acknowledge the financial support from CNPq/INCT and Faperj. We also thank Marcia Valeria Teles Machado and Andrea Virgínia Machado for the excellent research assistance.

†FGV/EPGE Brazilian School of Economics and Finance (e-mail: bruno.delalibera@fgvmail.br).

‡FGV/EPGE Brazilian School of Economics and Finance, FGV Crescimento & Desenvolvimento (e-mail: pedro.ferreira@fgv.br).
1 Introduction

Several recent studies on educational investments in early childhood have shown that expenditures in this stage of life increase the cognitive and non-cognitive development of children and increase the investment return in later stages of their lives. For instance, Rolnick and Grunewald (2003) and Schweinhart (2004) show that the private return on investments in preschool is high as it increases the marginal productivity of individuals. However, they show that there is also a sizable external effect due to improvement in the socioeconomic conditions of these persons in the form of, for instance, less crime, less time in jail and less use of social services.

Heckman and Carneiro (2003) assess the Perry Preschool Program\(^1\) and find that, when measured through age 27, the program returns $5.7 for every dollar spent. When returns are projected for the remainder of the lives of program participants, the return on the dollar rises to $8.7. Also analyzing the Perry Preschool Program, Heckman et al. (2010) document that the social return of the program is between 7% and 10%, which is considered high compared to other investments. The education, skills and competences acquired at this stage of life facilitate learning throughout the rest of the student’s life. Other studies emphasize that the early years are crucial for the formation of brain connections that capture the different impulses of the environment in which the child is located, affecting their intellectual development, personality and social behavior (Young and Mundial, 1996; Myers and de San Jorge, 1999; Knudsen et al., 2006; Irwin et al., 2007). According to Heckman (2011), the economic and intellectual inequality among individuals begins in early childhood because different investments in this stage lead to inequality in cognitive and non-cognitive skills in adulthood.

This article studies the impact of early childhood skill formation on the evolution of labor productivity and human capital accumulation in the U.S. between 1961 and 2008, a period in which pre-primary education went from practically nonexistent to enrollment rates close to 75%. We add early childhood human capital to a standard continuous time life-cycle economy and assume complementarity between the two types of educational stages, in accordance with the evidence from the empirical literature (Heckman and Cunha, 2007; Cunha et al., 2010). In the model, there are three sectors: goods, early childhood and formal education. Agents are homogenous and choose the intensity of preschool education\(^2\), how long to remain in formal schooling\(^3\), labor effort and consumption. Individuals do not work when they are in school. Retirement and the early childhood timespan are exogenous.

The model is calibrated to reproduce the American economy in 2008 and the trajectory of key variables since 1961. Some variables, such as distortions of the prices of early childhood education, formal education, labor and capital, are calibrated to match targets of U.S. data using the simulated method of moments. The model fits the data very well and closely reproduces the paths of schooling, hours worked, relative prices and GDP. We use the model to estimate endogenously early childhood education from 1961 to 2008, circumventing the lack of data, especially in the initial years.

Early childhood education plays a central role in our results. Although the estimated value of its weight in the human capital function is very low, the complementarity in human capital formation

\(^1\)The Perry Preschool Program, conducted in the United States in the 1960s, treated for two years a group of poor 3-year-old children. This treatment included daily classes and weekly visits of the teachers to the students’ homes. The program collected data from this treatment group and a control group and followed them until they were 40 years old.

\(^2\)In this paper, we use early childhood education, preschool education and early childhood skill formation as synonyms.

\(^3\)In this paper, we use formal education, late education and years of schooling as synonyms.
between both stages of education ultimately amplifies the impact of early childhood education through its effect on formal schooling. That is, a rise of early childhood education and other forms of skill formation at this age positively affects the return to formal education, leading to an increase in schooling. In our simulations, we find that the expansion of early childhood education in the 1960-2008 period could explain approximately 60% of the observed increase in schooling in the U.S. Moreover, it is as important as formal education in explaining GDP per capita and productivity.

Our formulation of early childhood human capital is quite general. It can be considered as a reduced form specification that encompasses different factors that affect the cognitive and noncognitive skill development of younger children. The literature emphasizes that parenting time, money expenditures, preschool education and different forms of early intervention are all important sources of learning and skill formation, and key to future economic and social success (e.g., Heckman (2000) and Heckman and Cunha (2007)). The Perry Program, the Abecedarian Program in North Carolina and the Chicago Child-Parenting Centers, for instance, had classroom curricula that were based on key factors of cognitive development related to planning, expression, understanding, etc. However, they also included visits to children’s home, whose purpose was to involve mothers in the educational process of their children.

Carneiro and Rodrigues (2009) and Agostinelli and Sorrenti (2018) using cross-section time surveys, find that parental time is a very important factor for child development. These articles, and many more, find that maternal labor supply has a negative impact on the skill formation of their children. Using longitudinal survey data from the UK Millennium Cohort Study, Bono et al. (2016) also estimates that maternal time is a quantitatively important determinant of skill formation and finds evidence of long-term effects of early maternal time inputs on cognitive skill development. Of course, not all time spent with the children has the same effect, watching TV together is not as good as reading and playing (Samuelsson and Carlsson, 2008; Swing et al., 2010), which means that the quality time parents spend with their offspring is more important for skill development.

It is not clear, however, how these factors have evolved over time. For instance, it is well documented that the labor force participation of woman increased significantly in the second half of the last century. However, we do not know if this has had any important effect on the time mothers spend with their sons and daughters. In a recent article, Cardia and Gomme (2018) using data from the American Time Use Survey shows that between 1965 and the average of the years 2003-2015 married woman in their prime childcare years (aged 24-29) decreased their primary childcare time (e.g., direct and exclusive activities) in the period, but the opposite happened for older woman. It is shown that along their entire lifetime the time mothers spend with their children increased in this period, but not by much.45

Our formulation is a simple way to circumvent this problem and allows us to work in a longer time span. The early childhood human capital variable used in the model represents the intensity of preschool education and of any other factor affecting skill formation at this age. There is no direct correspondence to any single variable such as time spent in school, money expenses or parenting (quality) time. It is measured endogenously from the model and for simplicity we call it early childhood education.

We extend the model in later sections by introducing school quality and a stylized form of parenting time in the human capital production function. Our main results are robust to these modifications. We also added, in another version of the model, government expenditures in both

---

4This is the only time series information on the subject we could find, and yet it has only two points in time.
5In the opposite direction, Ramey and Ramey (2010), when evaluating the Head Start Program, stress that in their opinion Head Start’s inability to engage the majority of parents meaningfully in their child’s education was the most frequent concern expressed by Head Start directors and teachers.
education. The model was recalibrated, and the main result here is that a small reallocation of educational resources for early childhood education (an addition of 0.3% of public expenditures) would have increased income per capita by 0.36%. This finding reinforces our previous results and calls for a reassessment of educational policies in favor of more attention and expenditures in the early stages of human capital formation.

This article extends and improves the previous literature in several directions. First, we relate to the literature that studies human capital accumulation in a dynamic macroeconomic framework (Rangazas, 2000, 2002; Lee and Wolpin, 2010; Restuccia and Vandenbroucke, 2013a,b; Castro and Coen-Pirani, 2014; You, 2014; Kong et al., 2018). None of these articles studies early childhood education. Kong et al. (2018) use a model of on the job human capital accumulation embedded in a college choice model to study the flattening of life-cycle earnings profiles. You (2014) use the Ben-Porath (1967) model of human capital formation and finds that one-fifth of U.S. labor quality growth between 1967 and 2000 was due to the rise in educational spending. The main factor in explaining educational attainment is the skill price. In Lee and Wolpin (2010) and Castro and Coen-Pirani (2014), skill price is also an important force in explaining educational attainment. Therefore, these papers address distinct but complementary sets of mechanisms to explain schooling attainment.

The second body of literature that relates to our study investigates the links among government incentives, preschool education and economic development. For instance, similarly to this paper, Abington and Blankenau (2013) and Blankenau and Youderian (2015) embody the results of Cunha et al. (2010) and Heckman and Cunha (2007) in their human capital accumulation function and study the reallocation of government resources between educational stages and how it could affect aggregate income. The first article is a theoretical study, and neither paper is concerned with the evolution of labor productivity across time, as we are.

Our paper is also related to the body of literature that studies cross-country income differences and human capital (Klenow and Bils, 2000; Erosa et al., 2010; Schoellman, 2012; Córdoba and Ripoll, 2013; Restuccia and Vandenbroucke, 2014; Schoellman, 2014; Manuelli and Seshadri, 2014). Although Manuelli and Seshadri (2014) and Córdoba and Ripoll (2013) include preschool education in their models, they do not consider the complementarity between early childhood and later education stages, as documented by Cunha et al. (2010) and Heckman and Cunha (2007). Moreover, their objective is different from ours. Manuelli and Seshadri (2014) investigate the role of human capital in explaining cross-country differences of output per worker, while Córdoba and Ripoll (2013) provide a theory that explains the cross-country distribution of average years of schooling. In contrast, we study the effect of early childhood skill formation on subsequent stages of human capital investment and on productivity, focusing on a single country (USA) over time.

Schoellman (2014) documents, using data of refugees living in the U.S., that adult outcomes are independent of age at arrival in the U.S., up to age 6. He interprets this finding, using a simple model of human capital accumulation, as evidence that the differences across countries in early childhood education are not important in explaining development differences. Del Boca et al. (2014) use a standard life-cycle model to study the child quality investment trade-off: Parents work more to have more money to invest in their children, but in turn, less time is allocated to child development. They find that for early investments, parental input is more important than monetary investments. Although it is clear that parental input is also important, we do not model explicit this dimension because in our model fertility is exogenous. However, as said before, our modeling

---

6 Castro and Coen-Pirani (2014) analyze the evolution of educational attainment using a model with an inter-cohort with heterogeneous learning ability, in which the variation in schooling is given by changes in skill price, tuition, education quality and ability.
strategy is a reduced form formulation that aim to measure all factors affecting early childhood skill formation.

This paper proceeds as follows. In Section 2, we present some stylized facts about evolution of early childhood human capital and preschool education. In Section 3, we present the model. In Sections 4 and 5, we describe the calibration strategy and report how well the model fits U.S. data, respectively. In Section 6, we discuss results regarding the effect of both education stages in the evolution of human capital, GDP and U.S. development. In Section 7 and 8, we modify the model, introducing public expenditures in education and quality of formal education, respectively. In Section 9, we analyze the robustness of our results with respect to parameter values. Section 10 concludes the paper.

2 The Evolution of Preschool Education

As stated by Vinovskis (1993), there were few efforts to provide preschool training during the first half of the twentieth century, and most of them did not last long or had limited impact. Moreover, during the 1950s and early 1960s, policymakers and the general public paid little attention to preschool education. Only in the sixties did early childhood development receive more attention from American society, not only in the education dimension but also in health, special education, research, etc. (Shonkoff and Meisels, 1990). In the early sixties studies by Benjamin Bloom, among others, it is argued that early childhood education is key to improving later performance in schools and this view became widely accepted among experts.

New social demands, evolution of scientific production and greater public interest in early childhood, gave this topic a greater role in the debate on socio-economic inequalities. As stated by Shonkoff and Meisels (1990, p.15), “these included President Kennedy’s interest in mental retardation, the political impact of the civil rights movement, and President Johnson’s commitment to wage war on the sources and consequences of poverty”.

From the mid-sixties on several programs were created aimed at increasing early childhood attention and care. Among them, the Perry Pre School Project and the Head Start Program, which were created in 1962 and 1965, respectively; the Public Law (PL) 94-142 enacted in 1975 that provided funds for states to care for children as young as 3 years old; and the Public Law (PL) 99-457 enacted in 1986 that reinforced incentives for states to provide services for 3 to 6-years-old children, and established a discretionary program providing service to newborn children up to 3 years of age.

The improvements were fast. For instance, in 1965, only eighteen states had public kindergarten, and in 1970, less than 80% of five-year-old children attended public kindergarten. In 2000, all the states funded public kindergarten, and 98% of five-year-old children attended it (Kamerman and Gatenio-Gabel, 2007). The Head Start Program, which started in 1965 as a summer program with 561,000 children enrolled, was transformed the next year into a nine-month program, and in 2015, it served nearly one million children. Since its inception, Head Start has served more than 33 million children, and funding per student in real value increased 6.46 times between 1965 and 2008 (Head Start Bureau, 2012).

Table 1 presents the evolution of the absolute number of students and the gross enrolment rate

---

7 The Head Start Program is a federally funded preschool program, largely half day, targeted on poor children and serving 3-4-year-olds primarily. It provides comprehensive education, health, nutrition, social and other services.

8 The United Nations Educational, Scientific and Cultural Organization (UNESCO), defines Gross Enrolment Ratio as the total enrolment within a country in a specific level of education, regardless of age, expressed as a percentage of
(in brackets) of pre-primary, primary and secondary education from 1970-2011. The progress of enrolment in pre-primary education was striking in that period, especially if one takes into account that the figures for 1960 were close to zero. Indeed, the number of children enrolled in pre-primary increased 2.09 times between 1971 and 2011. In absolute terms, total enrolment increased more in pre-primary than in primary and secondary education. Pre-primary education had 4,707,000 of new enrollments in the period, while primary and secondary had just 2,394,000 and 3,691,000, respectively. The increase in the gross enrollment rate is also impressive, rising from 37% in 1970 to twice that forty years later, considerably reducing the distance with respect to primary and secondary education. In summary, the evidence indicates, since the mid-sixties, rapid and continuous expansion of pre-school education. We will show later that this development has had a significant impact on formal schooling, human capital and labor productivity.

Table 1: Enrolment in pre-primary, primary and secondary education for different years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Pre - Primary (thousands)</th>
<th>Primary (thousands)</th>
<th>Secondary (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971</td>
<td>4300 (37.7%)</td>
<td>22037 (88.49%)</td>
<td>20593 (83.84%)</td>
</tr>
<tr>
<td>1981</td>
<td>5163 (52.54%)</td>
<td>20420 (98.7%)</td>
<td>21585 (89.53%)</td>
</tr>
<tr>
<td>1991</td>
<td>7300 (65.17%)</td>
<td>22429 (104.42%)</td>
<td>19270 (90.88%)</td>
</tr>
<tr>
<td>2001</td>
<td>7538.72 (63.22%)</td>
<td>25297.6 (103%)</td>
<td>23087.04 (93.95%)</td>
</tr>
<tr>
<td>2011</td>
<td>9007.2 (74.17%)</td>
<td>24431.66 (100%)</td>
<td>24214.3 (94.67%)</td>
</tr>
</tbody>
</table>

Source: UNESCO.
Note: Numbers in brackets are the gross school enrolment ratio

3 The Model

Our modeling strategy for early childhood human capital accumulation emphasizes the intensity of education and other factors that influence the skill formation at this age. The early education/skills variable, $x$, does not represent a single factor. It can be seen as a reduced form specification that encompasses a group of relevant variables that, in its turn, will be a component of a C.E.S. human capital function, along with formal schooling time. In later sections, we will use the model to estimate this variable.

We consider a simple model that can be used analytically to examine the impact of early childhood education, and its complementarity to formal education, on human capital formation and development. The key elements in this economy will be the intensity of childhood education and the optimal time to leave formal school and the dynamic consequences of these decisions on human capital and growth.

3.1 Production

There are three sectors in this economy: Sector one produces consumption and capital goods, sector two supplies early childhood education, and sector three supplies formal education. The output in the goods sector, $Y_1$, is a function of physical capital service, $K$, and skilled labor, $H_1$. the population in the official age group corresponding to this level of education. Thus, it is possible to have a percentage that is higher than 100%.
The technology of both educational sectors takes into account that the production of educational services is labor intensive. For instance, the share of gross capital formation in total government expenditure in education was only 9.89% in 2010, according to the Organization for Economic Co-operation and Development (OCDE). In this sense, it is assumed that output in educational sectors uses only skilled labor as input. We therefore have

\[ Y_i = \begin{cases} A_i K^\alpha (H_i)^{1-\alpha} & \text{if } i = 1 \\ A_i H_i & \text{if } i \in \{2,3\} \end{cases} \]  

(1)

where \( A_i \) is the sector’s total factor productivity, and \( \alpha \) is the capital’s share in output. Skilled labor is given by:

\[ H_i = L_i h_i \]

where \( L_i \) is raw labor, and \( h_i \) is the human capital of workers.

Let good 1 be the numeraire and \( q_2 \) and \( q_3 \) be the relative prices of early childhood educational services and schooling services. Given free mobility of factors across sectors, we have:

\[ (1-\alpha) A_1 \kappa^\alpha = q_i A_i = w_i, \ i \in \{2,3\} \]  

(2)

\[ \alpha A_1 \kappa^{\alpha-1} = R, \ \text{com} \ \kappa = \frac{K}{L_1 h_1} \]  

(3)

where \( R \) is the rental price of capital, and \( w \) is the wage rate of raw labor.

After algebraic manipulations, the per capita output supply of each sector is given by:

\[ y^s_i = \begin{cases} A_i \kappa^{\alpha} l h_i l_i & \text{if } i = 1 \\ A_i l h_i l_i & \text{if } i \in \{2,3\} \end{cases} \]  

(4)

where \( l \) is the number of workers per capita and \( l_i \) is the sector share of workers to the total number of workers in the economy.

### 3.2 Household

Time is continuous, and individuals live for \( T \) years. In the first part of the life cycle, \( T_i \), individuals stay in preschool and then go to school for \( T_s \) years. After leaving school, they join the labor market, and once they leave school, they cannot return. Retirement is mandatory, and active life ends after \( T_w \) years working.

Heckman and Cunha (2007) document that human capital consists of investments in different stages of life and that early childhood education is complementary to formal education. They show that for a disadvantaged child, the return is higher on the former than on the latter. In this sense, we follow Heckman and Cunha (2007) and use a CES function for human capital formation, taking into account these two stages of life \( (T_i \) and \( T_s \)). This formulation will be calibrated so that there is complementarity between the first stage \( (T_i) \), which is fixed at 6 years, and the second stage \( (T_s) \), which is an individual choice.

In the model, individuals begin the second stage of cognitive development with the human capital that they accumulated in the first stage. Total human capital will be a composite function of early childhood education, \( x \), and formal schooling, \( T_s \):
\[ h(T_s, x) = \theta \left( \lambda h_E(x)^\sigma + (1 - \lambda) T_s^\sigma \right)^{\frac{1}{\sigma}} \]  

(5)

where \( T_s \) is the time spent on primary, secondary and tertiary education and \( h_E \) is early childhood human capital, which is given by:

\[ h_E(x) = \int_0^{T_i} x \, dt = x T_i \]

In the above equations, \( \theta \) is a normalization parameter, \( \lambda \) is the weight of early childhood education on the human capital function, and \( \sigma \) is the parameter that characterizes the elasticity of substitution. It is easy to see that cross derivatives are positive, so that the higher \( x \) is, the longer individuals will stay in formal school\(^9\).

At each instant of time, individuals choose consumption. During the education period, they decide how much to invest in early childhood education, \( x \) (intensive margin), and schooling time, \( T_s \) (extensive margin). Then, in the next period, the household decides how much work effort to supply. The preference of a household is given by:

\[ \int_0^T e^{-\rho t} \ln(c(t)) \, dt + \beta \int_{T_i}^{T_y} e^{-\rho t} \ln(1 - l(t)) \, dt, \beta > 0 \]

(6)

where \( T_y = T_i + T_s, c(t) \) is the consumption in time \( t \), \( l(t) \) is the labor offer in time \( t \), and \( \rho \) and \( \beta \) are, respectively, the discount rate and the leisure preferences in terms of consumption.

The expenditure side of the budget constraint consists of the consumption and payment of both school tuitions over time. The revenue is composed of wages from labor services, rents from capital, and lump-sum transfers. Therefore, the intertemporal budget constraint is given by

\[
\begin{align*}
\int_0^T e^{-rt} c(t) \, dt + (1 + \tau_H) \int_{T_i}^{T_i + T_s} e^{-rt} x q_2 \, dt + (1 + \tau_Hc) \int_{T_i + T_s}^{T_i + T_s + T_y} e^{-rt} q_3 \, dt = \\
\int_0^T e^{-rt} \chi \, dt + (1 - \tau_L) \int_{T_s}^{T_y + T_w} e^{-rt} w(T_s, x) l(t) \, dt
\end{align*}
\]

(7)

where \( w(T_s, x) = w h(T_s, x) \) is the wage rate of a worker with \( x \) units of investment in early childhood education and \( T_s \) years of schooling; \( r \) is the interest rate; \( \tau_H \) and \( \tau_Hc \) are, respectively, the distortions or subsidy to formal and early childhood school tuition\(^{10}\); \( \tau_L \) is the distortion in wages; and \( \chi \) is government transfers.

The optimal consumption level is obtained by solving equation (6), subject to budget constraint (7). To simplify, we assume that the interest rate is equal to the discount rate, \( r = \rho \). With this assumption, the growth rate of consumption and labor supply is zero, so that the optimum paths of consumption and labor are constant throughout the life cycle\(^{11}\). Furthermore, there are two decisions concerning education. Individuals choose the optimal educational investment in the first

\(^9\)Note that early and late schooling investments are not symmetric in the way they enter into the human capital production function. This was done by convenience, as we assumed that the quantity margin of preschool is fixed, while formal years of schooling is not. This hypothesis is relaxed in Section 8, where we added an intensive/quality margin to late schooling. We also discuss in the appendix (section A.1) the evolution of the parental time invested in the education of their children and present a quantitative evaluation of the introduction of this variable in the human capital function.

\(^{10}\)In Section 3.5 we discuss the distortions.

\(^{11}\)The assumption that \( r = \rho \) is common in the literature. For instance, see Restuccia and Vandenbroucke (2013a).
period of their life, x, and then the optimal time, \( T_s \), to leave school. In the latter decision, individuals consider that the longer they stay in school, the shorter their productive life, \( T_W \), as retirement, \( T_R \), is exogenous. In this way, we have \( T_i + T_s + T_w = T - T_R \), which is independent of the schooling-years decision. In this case, the first-order conditions are:

\[
\begin{align*}
\bullet [l]: & \quad l = 1 - \frac{c\beta}{(1 - \tau_L)wh(T_s, x)} \\
\bullet [c]: & \quad c = (1 - \tau_L)wh(T_s, x)1 - e^{-(T_i + T_s)}\frac{1 - e^{-rT_w}}{1 - e^{-rT_i}} + (1 + \tau_H)xq_2 \frac{1 - e^{-rT_i}}{1 - e^{-rT_i}} - (1 + \tau_H)q_3e^{-rT_i}\frac{1 - e^{-rT_i}}{1 - e^{-rT_i}} \\
\bullet [T_s]: & \quad (1 - \tau_L)wh(T_s, x)\left(\frac{1 - e^{-rT_w}}{r}\right) = (1 - \tau_L)wh(T_s, x)l + (1 + \tau_H)q_3 \\
\bullet [x]: & \quad (1 - \tau_L)wh(T_s, x)\left(\frac{e^{-r(T_i + T_s)} - e^{-r(T_i + T_s + T_w)}}{r}\right) = (1 + \tau_H)xq_2 \frac{1 - e^{-rT_i}}{1 - e^{-rT_i}}
\end{align*}
\]

Equations (8a), (8b), (8c) and (8d) are the first order conditions with respect to labor, consumption, formal education and early childhood education, respectively. Equations (8a) and (8b) are common in the literature. For instance, in the labor equation the human capital term reflects the opportunity cost of leisure and at the same time positively affects consumption.

Given the early childhood investment, equation (8c) equates the marginal contribution of schooling to lifetime earnings, on the left-hand side, to its marginal cost, on the right-hand side. The latter is the opportunity cost of not working plus the tuition cost at the stopping time. The former is the impact on the present value of the time endowment of staying in school one additional unit of time, which implies higher wages in the future due to higher human capital. Equation (8d) has a distinct interpretation because the investment in early childhood is given in intensive terms, and there is no opportunity cost of time. Therefore, this equation equates the increase in earnings only to the increase in tuition costs.

### 3.3 Demography

At each instant, mass \( me^{nt} \) of homogeneous agents is born. As the total population in period \( t \) is the sum of all the generations living in \( t \), the total population is given by

\[
\int_{t-T}^{t} me^{na} da = m(\frac{1 - e^{-nT}}{n})e^{nt}
\]

where \( t - T \) is the oldest generation, and \( t \) is the newest generation.

Assuming \( m = \frac{n}{1 - e^{-nT}} \), the total population at \( t \) period is \( e^{nt} \), and the population growth rate is \( n \).

Let \( N(t) \) be the share in the total population in period \( t \). In this way,

\[
N(t) = \frac{m}{e^{nt}} = me^{-mt}
\]
Therefore, define

\[
N_i = \int_0^{T_i} N(t) dt = \frac{1 - e^{-n T_i}}{1 - e^{-n T}} \tag{9a}
\]

\[
N_s = \int_{T_i}^{T_i + T_s} N(t) dt = \frac{e^{-n T_i} (1 - e^{-n T_s})}{1 - e^{-n T}} \tag{9b}
\]

\[
N_w = \int_{T_y}^{T_y + T_w} N(t) dt = \frac{e^{-n T_y} (1 - e^{-n T_w})}{1 - e^{-n T}} \tag{9c}
\]

\[
N_R = \int_{T_y + T_w}^{T_y + T_w + T_R} N(t) dt = \frac{e^{-n (T_y + T_w)} (1 - e^{-n T_R})}{1 - e^{-n T}} \tag{9d}
\]

where \( N_i, N_s, N_w \) and \( N_R \) are the population of children, formal students, workers, and retirees as a share of the total population, respectively\(^{12}\).

### 3.4 Aggregate Consumption and Total Labor Supply

The total flow of labor services per capita in this environment is the sum of all individuals’ labor supply for those in the labor market. Therefore, using equations (8a) and (9c), we have

\[
I^s = \int_{T_y}^{T_y + T_w} N(t) I(t) dt = e^{-n T_y} \frac{1 - e^{-n T_w}}{1 - e^{-n T}} \left( 1 - \frac{\beta c}{(1 - \tau_L)wh(T_s, x)} \right) \tag{10}
\]

The aggregate consumption is the sum of all individuals’ consumption at instant \( t \). Therefore,

\[
C = \int_0^{T_i} N(t) c(t) dt + \int_{T_i}^{T_i + T_s} N(t) c(t) dt + \int_{T_y}^{T_y + T_w} N(t) c(t) dt + \int_{T_y + T_w}^{T_y + T_w + T_R} N(t) c(t) dt \tag{11}
\]

However, with the assumption \( \rho = r \), we have \( c(t) = c \) for all \( t \in \{0, T\} \). Therefore, \( C \) is given by equation (8b).

### 3.5 Policy Distortions

In this economy, there are distortions or subsidies to the choice of human capital, labor supply and capital. These distortions are represented by tax on early childhood education, formal education, labor effort and the rental rate of capital. Net tax revenue is equally distributed to individuals through a lump-sum transfer, \( \chi \).

Although several factors may cause these distortions, we consider a generic formulation that encompasses many possible distortions. For example, some factors that are not modeled here can be thought of as distortions in the choice of human capital. Among them, we highlight credit constraints, which are an important factor in educational choices, particularly for poor families; information problems regarding the importance of early childhood education in the formation of children’s cognitive development; myopic behavior; and the lack of altruism (Restuccia and Urrutia, 2004; Heckman and Cunha, 2007; Córdoba and Ripoll, 2013; Attanasio, 2015). Thus, the distortions here are generic factors causing the misallocation of resources.

\(^{12}\)Note that the sum \( N_i + N_s + N_w + N_R = 1 \).
Therefore, the per capita transfers is the sum of all revenue weighted by the share of total population $e^m$. Thus, per capita transfers are

$$
\tau_H x q_2 N_i + \tau_H q_3 N_s + \tau_L w h(T_s, x) N_w + \tau_K R \kappa = \chi
$$

where $N_i$, $N_s$ and $N_w$ are given by demography equations, and $k$ is the per capita capital of economy.

### 3.6 Equilibrium

In early childhood education, individuals consume $x$ units of education, whereas in formal education, they consume 1 unit of education. Thus, with the demography structure in mind, we have

$$
y_2^d = \int_0^{T_i} x N(t) dt = x N_i
$$

$$
y_3^d = \int_{T_i}^{T_i+T_s} N(t) dt = N_s
$$

where $N_i$ and $N_s$ are given by equations (9a) and (9b), respectively.

The aggregate demand of sector 1 is given by the sum of capital and consumption aggregate demands:

$$
y_1^d = C + (\delta + n) k
$$

where $C$ is given by equation (11) and $k$ is the capital per capita.

The steady-state equilibrium of this economy is given by prices $\{q_2, q_3, r, w\}$, government transfers $\chi$ and allocations $\{c^*, l^*\}$, $\{T_s^*, x^*\}$ and $k^*$, such that

i) Given a list of distortions $\{\tau_H, \tau_H, \tau_L, \tau_K\}$, prices $\{q_2, q_3, r, w\}$ and transfers $\chi$, $\{c^*, l^*\}$ and $\{T_s^*, x^*\}$ are optimum allocations of the consumer problem.

ii) Given a list of prices $\{q_2, q_3, r, w\}$, $k^*$ is an optimum allocation of the problem of the firm.

iii) Net tax revenue is equally distributed to individuals through a lump-sum transfer, $\chi$.

iv) The equilibrium in the educational market is:

$$
y_2^s = x N_i
$$

$$
y_3^s = N_s
$$

v) The equilibrium in the goods market is:

$$
y_1^s = C + (\delta + n) k
$$

vi) The equilibrium in the asset market is:

$$
r = (1 - \tau_k)R - \delta = (1 - \tau_k)\alpha A_1 k^{\alpha-1} - \delta
$$
4 Calibration

Our calibration strategy involves choosing parameters so that the steady equilibrium of the model are consistent with observations for the United States. We compute the equilibrium and perform the calibration for the following years\(^\text{13}\): \{1961, 1970, 1980, 1990, 2000, 2008\}. In other words, we assume that the economy is in a different steady state in each one of these years and calibrate accordingly.

There is a group of parameters that are constant over time and are observed in the data. We follow the standard procedure of employing data from national accounts, the U.S. Census and values commonly used in the literature. Table 2 presents these parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n)</td>
<td>(\log(1.0125))</td>
<td>U.S. Census</td>
</tr>
<tr>
<td>(\delta)</td>
<td>(\log(1.066))</td>
<td>NIPA</td>
</tr>
<tr>
<td>(r)</td>
<td>(\log(1.04))</td>
<td>Standard</td>
</tr>
<tr>
<td>(\rho)</td>
<td>(\log(1.04))</td>
<td>Assumption (\rho = r)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.4</td>
<td>NIPA</td>
</tr>
<tr>
<td>(T_i)</td>
<td>6</td>
<td>Standard</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>-0.5</td>
<td>Heckman et al. (2010)</td>
</tr>
<tr>
<td>(\theta)</td>
<td>1</td>
<td>Normalization</td>
</tr>
</tbody>
</table>

The population growth rate, \(n\), is the average U.S. population growth between 1900 and 2010, obtained from the *U.S. Census*. The National Income and Product Accounts (NIPA) provides the depreciation rate and the capital share in the goods sector, which are set to 6.6% and 40%, respectively\(^\text{14}\). The interest rate is set at 4\%, and the parameter \(\rho\) has the same value. The first stage of the life cycle, \(T_i\), is the pre-school period and is set at 6.

When studies on economic growth disregard early childhood education in human capital accumulation, they implicitly assume that the two human capital stages are substitutes. However, according to Heckman and Cunha (2007), an optimal allocation of human capital resources would be a distribution of investment between these stages. Thus, there is some degree of complementarity in equation (25); i.e., parameter \(\sigma\) should be negative. Cunha et al. (2010) confirm the conclusion of Heckman and Cunha (2007). Indeed, with a nonlinear factor model with endogenous input, Cunha et al. (2010) estimate that the stages of human capital formation are complementary. Thus, the parameter that characterizes the complementarity of human capital function, \(\sigma\), is set at \(-0.5\), which is an intermediate value of Heckman et al. (2010) estimation.

Our preferred calibrated value of \(\sigma\) is also close to that used in Cunha et al. (2006), -0.4108, in a simulation of the costs of remediation of late versus early and late intervention. This value, in its turn, is close to that Heckman and Cunha (2007). Note, however, that Agostinelli and Wiswall (2016) when analyzing the implications of some of the assumptions in the estimation approach of Heckman and Cunha (2007) and Cunha et al. (2010) find that they can bias the estimates of the production technology. In particular, they suggest that it can bias the estimates of \(\sigma\) toward zero. In

\(^{13}\)Data restrictions explain the choice of 1961 as the first year. The last year is 2008 to avoid the financial crisis and its consequences.

\(^{14}\)For the depreciation rate, we use the long-run average for the investment/capital ratio.
this sense, it is quite important to study the sensitivity of the simulation results to modifications in the values of this parameter. This is done in Section 9, where we simulate the model for different values of $\sigma$ ($\sigma \in \{-1, -0.75, -0.25, -0.01, 0.01\}$) re-calibrating the model for each different $\sigma$. We think that this set of parameter values safely includes most of the estimates in the literature. As we will see, the main results do not change much in most cases.

Demographic structure in this economy is given by life expectancy and retirement. Life expectancy at birth $T$ for 1961, 1970, 1980, 1990, 2000 and 2008 is set to 70.27, 71.11, 74.01, 75.37, 76.74 and 77.94, respectively, which are the values reported in the World Development Indicators database. Additionally, the age of retirement is set to 61, 65, 63, 62, 63 and 63.45 for 1961, 1970, 1980, 1990, 2000 and 2008, respectively, which are the values reported in the Current Population Survey, 1962-2012, of the U.S. Bureau of the Census for the men. Thus, the retirement time is the gap between life expectancy at birth and the age of retirement.

The remaining parameters to be calibrated are:

$$\{A_1, A_2, A_3, \tau_K, \tau_L, \tau_H, \lambda, \beta, \}$$

For each year, we have different distortions and TFP parameters but the same parameters of technology, $\lambda$, and preference, $\beta$. Thus, in total, we have 44 parameters to calibrate. To reduce the number of parameters and simplify the calibration procedure, we normalize $A_2$ to 1 and assume that some parameters have a constant growth rate. Thus, we set

$$\xi(s) = \xi^{2008} e^{g(s-2008)}$$

where the $g$ superscript is the growth rate and $s$ is the year. With this simplification, we still need to calibrate 13 parameters:

$$\theta = \{A_1^0, A_1^g, A_3^0, A_3^g, \tau_L^g, \tau_L^0, \tau_H^g, \tau_H^0, \tau_{Hc}^g, \tau_{Hc}^0, \tau_K, \lambda, \beta, \}$$

These parameters are calibrated so that in equilibrium, the model economy matches the following targets from the U.S. data:

i) GDP per capita in 2008 that is normalized to 1.

ii) Investment share of GDP in 2008 set to 0.2208 (Penn World Table).

iii) Share of early childhood sector workers in 2008 set to 0.0034 (OCDE statistics).

iv) Share of formal school sector workers in 2008 set to 0.0378 (OECD statistics).

v) Share of average worked hours in 2008 set to 0.3067 (OECD statistics).

vi) Average years of schooling in 2008 set to 13.2867 (Barro and Lee database).


---

[15] In the appendix (section A.2) we relax this hypothesis and our calibration and main results are robust.


xiii) Mean of GDP per capita growth rate between 1961 and 2008 set to 2.25% per year (World Development Indicators)

In this model, the output per capita of the economy is given by:

\[ y = y_1 + q_1 y_2 + q_2 y_3 \]  

(20)

Thus, we use \( y \) to match GDP per capita. The investment share of GDP is equal to \((\delta + n)k\), and the share of workers in each sector\(^{16}\) is given by equation (4).

As agents are homogeneous in our model, the average years of schooling is the optimum value of \( T_s \) that is implicitly set in the schooling decision, given by equation (8c) and (8d).

The life-cycle average labor supply of an individual is given by the labor supply throughout the life-cycle divided by the time that the individual could allocate to work. Therefore,

\[ \bar{l} = \frac{1}{T - T_s} \int_{T_s}^{T_s + T_w} l(\varepsilon) d\varepsilon \]

This statistic matches the average hours worked. For this, we assume that there are a total of 16 hours of discretionary time per day, which implies a total of 5840 (365*16) hours per year. Therefore, in 2008, we find that this statistic targeted a 1791/5840 ratio, where 1791 is the average hours worked in 2008.

Formally, the calibration strategy can be described as follows. Given a value for \( \theta \), we compute an equilibrium for each year \( s \in \{1961, 1970, 1980, 1990, 2000, 2008\} \) and define the following objects:

\[
J_1(\hat{\theta}) = \begin{bmatrix}
V(y_s(\hat{\theta})) - 0.0636 \\
E(y_s(\hat{\theta})) - 0.6763 \\
V(l_s(\hat{\theta})) - 9.185E-05 \\
E(l_s(\hat{\theta})) - 0.3171 \\
V(T_s(\hat{\theta})) - 2.4782 \\
E(T_s(\hat{\theta})) - 11.6987
\end{bmatrix}
\]

\(^{16}\)It is represented by \( l_i \) for \( i \in \{1, 2, 3\} \).
where $V(.)$ and $E(.)$ indicate the variance and mean for years \{1961, 1970, 1980, 1990, 2000, 2008\}. The second object that we define is

$$J_2(\hat{\theta}) = \begin{bmatrix} y_{2008}(\hat{\theta}) - \epsilon_{0.0225(46)} \\ y_{1961}(\hat{\theta}) - \epsilon \\ y_{2008}(\hat{\theta}) - 0.3067 \\ \frac{1}{T_{2008}}(\hat{\theta}) - 13.2867 \\ \frac{1}{T_{2}}(\hat{\theta}) - 0.0034 \\ \frac{1}{T_{3}}(\hat{\theta}) - 0.0378 \\ \frac{1}{T_{3}}(\hat{\theta}) - 0.2208 \end{bmatrix}$$

where the superscript indicates the year in which the respective variable was taken.

The first component of this vector indicates that between 1961 and 2008, the GDP per capita grew at a rate of 2.25% per year. The other terms are GDP per capita, life-cycle labor supply, years of schooling, share of workers in the secondary and tertiary sectors and investment/GDP ratio, in this order.

Then, to find $\theta$, we solve the following minimization problem using the simulated method of moments with the identity matrix as the weighting matrix:

$$\min_{\theta} J_1^\top(\theta)J_1(\theta) + J_2^\top(\theta)J_2(\theta)$$

Table 3 summarizes the results of our endogenous calibration. The model matches the targets very well. For instance, GDP per capita in 2008 is the worst match\(^\text{17}\) but in this case, the error is only 2%. Furthermore, the objective function value of the minimization problem is 8.2E-04, and the average percentage error – the gap between the U.S. data and the model – is 0.48%. Thus, the model reproduces the U.S. data very well.

### Table 3: Endogenous parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>U.S. data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leisure preference</td>
<td>$\beta$</td>
<td>0.65</td>
<td>0.317</td>
<td>0.316</td>
</tr>
<tr>
<td>Weight of early childhood in Human Capital Technology</td>
<td>$\lambda$</td>
<td>0.013</td>
<td>11.7</td>
<td>11.67</td>
</tr>
<tr>
<td>Initial Labor Tax</td>
<td>$\tau^0_L$</td>
<td>0.7</td>
<td>0.307</td>
<td>0.308</td>
</tr>
<tr>
<td>Capital Tax</td>
<td>$\tau_K$</td>
<td>0.45</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>Initial Schooling Tax</td>
<td>$\tau^0_H$</td>
<td>0.6</td>
<td>13.29</td>
<td>13.33</td>
</tr>
<tr>
<td>Initial Early Childhood Education Tax</td>
<td>$\tau^0_{HC}$</td>
<td>1.21</td>
<td>0.0034</td>
<td>0.0034</td>
</tr>
<tr>
<td>TFP Initial Goods sector</td>
<td>$A^0_1$</td>
<td>0.42</td>
<td>1</td>
<td>1.02</td>
</tr>
<tr>
<td>TFP Initial Schooling sector</td>
<td>$A^0_3$</td>
<td>2.3</td>
<td>0.0378</td>
<td>0.0378</td>
</tr>
<tr>
<td>Trend of TFP goods sector</td>
<td>$A^t_1$</td>
<td>0.0078</td>
<td>0.0636</td>
<td>0.0629</td>
</tr>
<tr>
<td>Trend of TFP schooling sector</td>
<td>$A^t_3$</td>
<td>-0.0089</td>
<td>0.0225</td>
<td>0.0228</td>
</tr>
<tr>
<td>Trend of labor tax</td>
<td>$\tau^t_L$</td>
<td>-0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>Trend of schooling tax</td>
<td>$\tau^t_H$</td>
<td>-0.024</td>
<td>2.478</td>
<td>2.477</td>
</tr>
<tr>
<td>Trend of early childhood education tax</td>
<td>$\tau^t_{HC}$</td>
<td>-0.06</td>
<td>0.676</td>
<td>0.678</td>
</tr>
</tbody>
</table>

Note: This sample is given by \{1961, 1970, 1980, 1990, 2000, 2008\}.

\(^{17}\)Remember that GDP per capita is defined by equation 20.
The fact that the U.S. economy is assumed to be in steady state every ten years does not affect our results related to the effects of education and human capital on the economic variables. All the data used in the calibration refer to the productive sector of the economy. In fact, the data on years of schooling is the only one that would not clearly represent the productive sector, but the average years of schooling of the labor force is calculated only for individuals over 25 years of age (i.e. for individuals who are of working age). Thus, any modification in the educational policy has an immediate impact in the model on the productivity of individuals, for instance 18.

5 Results

Figure 1 shows the results for schooling, output and hours worked. In this simulation, we use the calibrated parameters to compute the equilibrium for each year between 1961 and 2008. The dynamics of the model are given by changing parameters \( \{A_1, A_3, \tau_L, \tau_H, \tau_{Hc}\} \) and the demographic distribution, which is given by retirement age and life expectancy at birth.

![Figure 1: Model and U.S. data - 1961 to 2008](image)

(a) Years of schooling  
(b) GDP per capita  
(c) Average Hours of Work

The blue and red lines are respectively the model’s results and U.S. data. The model results are very close to the U.S. data. The average model errors in absolute terms for school, GDP and hours worked\(^{19} \) are 0.76%, 1.92% and 1.43%, respectively \(^{20} \). It is noteworthy that the fit for schooling and hours worked is surprisingly good because we use little information for the first years of these series. Although we do not use information from 1961 as a target, in the 1960s the average errors

---

18 A possible problem is that by using contemporary teacher labor force (which increased in time) to calibrate preschool education, we may overestimate \( x \). To check if this is an issue, we run the model with values of \( x \) lower than the estimated values. The main results (section 6) did not change significantly, i.e., the evolution of early childhood education is the main force to explain the evolution of income per capita and years of schooling.

19 Note that the model average hours of work is less smooth than the data. This due to the fact that retirement age, \( T_R \), and life expectancy, \( T \), are exogenous and change every year.

20 The variances of these errors are 6.14-E05, 2.66-E04 and 7.92-E05, respectively.
in absolute terms were 0.23\% and 1.96\%, for school and hours worked, respectively. Furthermore, for 1961, the first year of our series, the average errors for these series were 0.68\% and 1.87\%, in this order.

As a robustness check of our calibration, we also calibrate and simulate the model for other two sets of years: i) $Set_1 = \{1961, 1962, 1963, 2006, 2007, 2008\}$; and ii) $Set_2 = \{1961, 1965, 1970, 1980, 2000, 2005, 2008\}$. Table 10 in the appendix shows that the parameters are very close to the benchmark calibration, and a simulation shows that the model is still a good fit to the U.S. data. Indeed, the average error in absolute terms for schooling, GDP and hours worked are respectively 0.78\%, 1.99\% and 1.42\% for $Set_1$ and 0.89\%, 1.83\% and 1.42\% for $Set_2$.

Figure 2 presents the estimation for early childhood education. This variable does not have a direct counterpart in the data, and was generated endogenously in the model. This variable shows a large increase between 1961 and 2008. Indeed, early childhood education in the model increased 6.46 times between 1961 and 2008, and its share of human capital increased 3.88 times. Although it is not directly comparable to the statistics in Table 1, it mimics the observed increase in pre-school education in the period.

![Figure 2: Early Childhood Education - Calibrated model](image)

Return to education is another dimension of possible validation of the model. In the period, the average return for one additional year of education in the model economy was estimated to be 7.83\%. This value is close to the 1976-1995 average of the figures for the U.S. in Table A.4 in Psacharopoulos and Patrinos (2004). Of course, a representative agent model such as ours will have a hard time in closely reproducing education returns, as it varies considerably across groups and time. In our case, the returns fall in the period. Goldin and Katz (2008) estimate that the return for an additional high school year increased in the beginning of the same period, but falls after 1995. In absolute values, the figures are very close to ours. In the case of college, however, the trend is positive from 1979 to 2009 and above our estimates.\textsuperscript{21}

5.1 Relative Prices

We observe in the model a rise in the relative prices of education which is supported by U.S. data. The main mechanism is the relative increase in the labor productivity of the goods sector. The estimated reduction of the TFP of sector 3 and the increase in sector 1 over time help explain the increase\textsuperscript{22}.

\textsuperscript{21}In order to match these figures we would need heterogeneity in education across agents and some form of skill premium shock.

\textsuperscript{22}This is supported by the results in Hanushek and Rivkin (1997). Indeed, Hanushek and Rivkin (1997, p.37) state, “The consideration of expenditure in education or other labor-intensive activities typically refers to the cost implications
Figure 3: Model and Data prices - 1978 to 2008

(a) Model prices

(b) Prices index for U.S. economy - Bureau of Labour Statistics

Figure 3 shows the relative prices of the model and the price index for the U.S. economy. To relate the prices of our model (Figure 3a) with the data prices (Figure 3b), note that the variables "all items", "durable goods" and "housing" are associated with the goods sector, whereas the variables "services", "educational books and supplies", "tuition other school fees and childcare", "college tuition and fees and elementary" and "high school tuition and fees" are associated with the education sector. It is clear from Figure 3b that the prices associated with the education sector increased faster than the prices associated with the goods sector, which supports the results of our model. Indeed, in the U.S. data, the index prices of "all items", "durable goods" and "housing" increased 3.3, 1.62 and 3.46 times, respectively, whereas the index prices of "services", "educational books and supplies", "tuition, other school fees and childcare", "college tuition and fees and elementary" and "high school tuition and fees" increased 4.2, 7.3, 8.73, 9.55 and 9.62 times, respectively. In our model, the relative prices of formal and early childhood education increased 93.07% and 47.9%, respectively.

We conduct an experiment with the model with TFP parameters to measure the importance of the relative prices in our quantitative results. Basically, we keep the values of the TFP parameters ($A_1$ and $A_3$) constant at their 2008 levels, changing the others.

When we keep constant the TFP parameters at their 2008 levels, the prices of education for the other years is higher than in the calibrated model. This increased price would have a substantial impact on educational choices. For all years, formal education and early childhood education would be lower in the experiment. In 1961, when we have the largest gap between the experiment and the calibrated model, formal and early childhood education would be 60.48% and 55.18%, respectively, below the results of the benchmark model.23

In Restuccia and Vandenbroucke (2013a), the growth of wages, which is given exogenously, is the main factor explaining the evolution of years of schooling in the U.S. economy. In our model, however, the growth of wages is given endogenously, and TFP parameters are the exogenous force affecting the wages. What is important in our model for schooling choices is the growth of wages of differential technological change and productivity growth, a discussion that implicitly assumes efficient choices. The well-known theory, originally suggested by Scitovsky and Scitovsky (1959) and subsequently developed by Baumol and Bowen (1965) and Baumol (1967), concentrates on the cost disadvantages of a sector, of which education appears to be a prime example, that experiences little technological change while other sectors undergo regular productivity improvements. Because wages rise roughly in proportion with the average growth rate of labor productivity in all sectors, the technologically stagnant sector faces increased labor costs.”

23In 1961, the TFP experiment increased the prices of early childhood education and formal education by 1.85 and 2.8 times, in this order, with respect to the benchmark model.
relative to tuition costs, \( w_{qi} \), for \( i \in \{1, 2\} \). In this way, the impact of the TFP of the goods sector on schooling is insignificant because it affects only the level of wages and does not affect the ratio of wages to tuition costs.

6 Economic Development and Education

The theory we propose is convenient to study the relative contribution of early childhood and formal schooling to the variation, in the recent past, of human capital and income per capita in the U.S. The model includes, in addition to the two education decisions, saving and labor decisions and three sectors of production. In this general equilibrium framework, there are transmission mechanisms that are not generally present in the literature. For instance, early childhood decisions do not directly affect the decision to join the labor market; consequently, one could think that its impact on the income path along the life-cycle is not large. However, it indirectly affects the return to formal schooling – they are complementary – and hence individuals’ decision to remain longer in school and consequently its total effect on human capital and labor productivity can be quite high.

In the exercises presented in Figure 4, we simulate the model holding constant at its 1961 level one of the two educational stages. In this way, we can isolate and measure the effect of early childhood education or formal education on endogenous variables.

Figure 4: Constant level at 1961 for education - 1961 to 2008

Panel 4a presents the calibrated path of the benchmark calibration and simulated path of formal schooling – years of education – when early childhood education is kept constant at its initial level (the blue and green lines, respectively). Note that the graph shifts down markedly in the counterfactual simulation, and formal education by the end of the period would increase just 17.7%.

\[ 24 \text{See equation (8c).} \]
had early childhood education remained constant, in contrast to the observed increase of 45.3%. Thus, according to our model, 61% of the variation of schooling in the U.S. from 1961-2008 can be explained by the expansion of early childhood education alone.

From Figure 4b, one can see that the impacts of formal years of education and early childhood education on human capital were almost the same in the period: Had either of them remained constant, human capital would have been 31% lower than observed. This result is surprising given that the weight of early childhood education in the human capital function, \( \lambda \), is very small – only 0.013 – and the schooling weight is, consequently, 0.987. Although this implies that the direct return on early childhood investment is small, the complementarity in human capital formation of both types of educational stages ultimately cause the total effect of early childhood education on human capital to be very high. Increases in \( x \) and \( h_E \), as seen from the human capital function, boost the gains of schooling and hence leads to more years of education, further increasing human capital.

Early childhood education is as important as formal education in explaining the evolution of GDP per capita in the period, as seen in Figure 4c. Indeed, if in 2008 early childhood were at the same level as it was in 1961, income per capita would be 29% below its observed level, while it would decrease 28% if we had kept years of schooling at the 1961 level. This is not surprising after one considers that the effect of both educational stages in human capital (Figure 4b), and hence on labor productivity, are similar in magnitude. The impact of early childhood on labor productivity is high due to its interaction with formal education, so that the expansion of early childhood in the period can explain a large part of the increase in GDP per capita and labor productivity.

We also conducted an experiment to evaluate the contribution of life expectancy. Although life expectancy affects schooling, its contribution is not too large. For instance, if in 2008 life expectancy had been 70.27 years – the 1961 level – schooling would have been 13.09. Thus, life expectancy explained 6.07% of the observed increase in years of schooling in the U.S. This result is consistent with the findings of works such as Hazan (2009) and Restuccia and Vandenbroucke (2013b), who also find that life expectancy is not an important exogenous force to explain schooling evolution.

7 Introducing Public Expenditures on Education

One interpretation of the distortions/incentives for human capital accumulation is that they are a measure of, among other things, the impact of government policies. It would be interesting, hence, to incorporate a direct role of the government in affecting education. This is done in this section by adding public investment to the human capital functions (equation (25)).

We consider that investment in education now has private and public components. The human capital function of early education is now given by:

\[
h_1(x, g_1) = \left( (xT_i)^{\eta} + (g_1T_i)^{\eta} \right)^{\frac{1}{\eta}}
\]

(21)

where \( g_1 \) is government investment in early education and \( \eta \) is the parameter that characterizes the elasticity of substitution between private and public goods\(^{25}\). The human capital in the second

\(^{25}\)This formulation of human capital is similar to Blankenau and Youderian (2015); thus, we set \( \eta = 0.48 \), which is the calibrated value from this article.
stage is now given by:

\[ h(T_s, x) = \left( \lambda h_1(x, g_1)^\sigma + (1 - \lambda) h_2(T_s, g_2)^\sigma \right)^{\frac{1}{\sigma}} \]  \tag{22}

where \( h_2 \) is:

\[ h_2(T_s, g_2) = \left( T_s^{\eta} + (g_2 T_s)^{\eta} \right)^{\frac{1}{\eta}} \]  \tag{23}

In the function above, \( g_2 \) is government investment in formal education. Note that as in the previous case, the human capital accumulated in the first stage is a component of the human capital function of the second stage. Equation (12), the distortions/budget constraint equation of the government, is modified to:

\[ \tau_L w_l h(T_s, x) N_w + \tau_K R_k + (\tau_H c_x - g_1) q_2 N_i + (\tau_H - g_2) q_3 N_s = \chi \]  \tag{24}

There are no data on government expenditure on pre-primary education before the nineties; as a result, we estimate this series. We first calculate the average rate of the share of government expenditure on pre-primary to pre-primary through secondary education between 1998 and 2008, which is available in the UNESCO database. We then use pre-primary through secondary education expenditures as percentage of GDP, available in the US Government Spending database, for the years prior to 1998 and keep constant the estimated share of government expenditure on pre-primary education to estimate the government expenditure on pre-primary education between 1971 and 1997. Finally, we use student enrollment in both pre-primary and primary + secondary education, also available in the UNESCO database, to calculate the ratio of government expenditure per student to GDP per capita in each educational stage.

Figure 5 presents the ratio of government expenditure on primary through secondary to pre-primary education between 1971 and 2008. The red and blue lines are respectively the ratio of gross and per student expenditures. As we can see, government expenditure on primary through secondary was approximately 10 times higher than government expenditure on pre-primary. However, when we look for government expenditure per student, this ratio is less than 2. Indeed, the highest value that we observe of the per student series is 1.88 times, while the minimum value of the gross series is 9.30 times.

We calibrate this model to fit the U.S. economy following the previous procedure discussed in the calibration section\textsuperscript{26}. Table 6 summarizes the result. As before, the model matches the targets very well. GDP per capita in 2008 is the worst match; however, in this case, the error is only 1.05%.

Note that the calibration is not too sensitive to these changes. Indeed, only the distortion of formal education and the weight of early childhood education in the human capital function (\( \lambda \)) change significantly. This occurs because the government expenditure per student on late childhood education increased over time relative to early childhood education. Therefore, distortions in late childhood education capture this fact and, thus, is lower than in the benchmark model. In the same way, the parameter \( \lambda \) is higher because there is relatively more investment in formal education.

The largest share of total government expenditure on pre-primary education was observed in 2000, which was 9.7% of government expenditure on pre-primary, primary and secondary education. The lowest was 9.27% in 1971. To evaluate the impact of marginal increases in public

\textsuperscript{26}Due to data restrictions, we use only the sample \{1971, 1980, 1990, 2000, 2008\}.
expenditures on early education vis-à-vis formal schooling, we simulated the economy, keeping constant total government expenditure on education but increasing by 10%, for all years, the share of total government expenditure on pre-primary education.

In this experiment the government would spend, on average, 10.36% more on pre-primary students and 0.99% less on late childhood students. Income per capita would be higher in all years, 0.36% greater on average. This result reinforces the importance of early childhood education and its effect on labor productivity that was found in previous exercises.

Following Blankenau and Youderian (2015), we change (and recalibrate) our model to use in the human capital function, as they do, government expenditure on education as a percentage of GDP instead of expenditure per student. Therefore, if in 2008 the government expenditure on pre-primary education had been 0.004 higher (+0.4% of GDP), income per capita would have been 8.62% higher, which is near what they find in their study. Moreover, if the government had reallocated spending such that it were equal among pre-primary, primary and secondary education, income per capita would have been 2.19% higher. This very strong result is due to their use of total public expenditure and disappears once one uses public expenditure per student.
8 Education Quality

In this section, we introduce in the model a measure of quality of formal education. Education quality is an important aspect in deciding how many years of schooling the individual will acquire, but for tractability issues this variable, for the case of formal education, was not included as a choice in the consumer problem. We add now a quality measure that although it is not a choice of the individual, will evolve exogenously according to some measures of school performance observed in the data.

The quality of formal education did not increase significantly between 1971 and 2008 according to the National Assessment of Educational Progress (NAEP) tests. In fact, Table 5 presents the average NAEP results of mathematics and reading for American students aged 9, 13, and 17.

<table>
<thead>
<tr>
<th>Mathematics</th>
<th>Old Tests</th>
<th>Revised Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age 9</td>
<td>219</td>
<td>219</td>
</tr>
<tr>
<td>Age 13</td>
<td>266</td>
<td>264</td>
</tr>
<tr>
<td>Age 17</td>
<td>304</td>
<td>300</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reading</th>
<th>Old Tests</th>
<th>Revised Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age 9</td>
<td>208</td>
<td>210</td>
</tr>
<tr>
<td>Age 13</td>
<td>255</td>
<td>256</td>
</tr>
<tr>
<td>Age 17</td>
<td>285</td>
<td>286</td>
</tr>
</tbody>
</table>

Note: National Assessment of Educational Progress (NAEP) long-term trend assessments in reading and mathematics are based on the performance of nationally representative samples of 9-, 13-, and 17-year-olds attending public and private schools.

Table 5: Average Results of National Assessment of Educational Progress

The mean grade in the reading exam increased only 5.77%, 1.96%, and 0.35% for students aged 9, 13, and 17, respectively. In the mathematics the mean performance increased 10.96%, 5.64% and 0.66% for students aged 9, 13 and 17, in that order. Thus, we will recalibrate the main model considering that the quality of formal education increased by 20% between 1961 and 2008, which is approximately double the highest increase displayed in Table 5.

The main modification of the model is the introduction of a quality measure of formal education so that the first and second stages of human capital formation are now symmetrical. Other features of the model, such as the individual’s and the government’s budget restrictions were also modified. The human capital equation is now given by:

\[
h(T,x) = \theta \left( \lambda (x T_i)^\sigma + (1 - \lambda) (z T_i)^\sigma \right)^{\frac{1}{\delta}}
\]

(25)

In the simulations we forced \( z \) to increase from 1 to 1.2 between 1961 and 2008. The model was recalibrated following the same procedures as before and fits the data very well. Regarding the parameters, the education wedges are higher in 2008 now than in the original model. This

\[27\] After 2004 the methodology of the test underwent a modification as it is shown in the table.
happens because in the new model the marginal return on investment in education is greater than in the benchmark model (remember that the quality of formal education is now greater) and then the distortions need to be higher so that demand for education (formal and pre-primary) does not grow, fitting the data. The weight of preschool education in the human capital function, $\lambda$, was found to be 38.46% higher than in the original model. This can be explained by the higher values of human capital in the second phase of human capital formation, for a given $T$, since $z$ is greater than one.

Figure 6 presents the results of the same counterfactual exercises of Section 6.

Figure 6: Constant level at 1961 for education in a model with education quality - 1961 to 2008

(a) Years of schooling  
(b) Human Capital  
(c) GDP per capita

The main results remain when the quality of formal education increases in the period. There are, however, minor quantitatively differences. In Figure 6a preschool education is maintained at its 1961 level. Now preschool education explains 78.3% of the variation in formal education, rather than 61% as in the original model. Given complementarity between the two stages of human capital, the reduction in preschool education, with the now higher quality of formal education, will induce agents to choose less time in school to decrease the amount spent on education. In the model, increasing the quality of formal education linearly increases the cost of an additional year of study but the increase of the return to schooling is smaller.

In Figures 6b and 6c it is also possible to observe that preschool education has a greater impact than formal education when compared to the original model. In fact, if education were maintained at 1961 levels, human capital and GDP would be 30.46% and 27.55% lower than in the calibrated model, respectively, while in the case of preschool education human capital and GDP would be 38.52% and 36.4% lower than in the calibrated model. These results are expected, since the weight of preschool education on human capital is now higher than in the benchmark model without quality of education. In the latter, the impact on GDP and human capital was about the same.

\[28\] For each time unit the individual stays in school he now spends $zq$ in "effective" formal education, i.e., quality adjusted education.
9 Sensitivity

In this section, we analyze the sensitivity of the results with respect to the complementarity parameter $\sigma$. Studies on human capital formation find that investments across stages of cognitive development are complementary; i.e., early investment increases the productivity of later investment (Cunha et al., 2006; Heckman and Cunha, 2007; Heckman et al., 2010). We simulate the model with the following values: $\sigma \in \{-1, -0.75, -0.25, -0.01, 0.01\}$, and recalibrate the model for each different $\sigma$. The parameters are shown in Table 6.

<table>
<thead>
<tr>
<th>Complementarity</th>
<th>$\beta$</th>
<th>$\lambda$</th>
<th>$\tau_1^H$</th>
<th>$\tau_K$</th>
<th>$\tau_H^0$</th>
<th>$A^0_1$</th>
<th>$A^H_1$</th>
<th>$A^L_1$</th>
<th>$\tau_1^L$</th>
<th>$\tau_H^L$</th>
<th>$\tau_H^G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma=-1$</td>
<td>0.64</td>
<td>0.003</td>
<td>0.71</td>
<td>0.45</td>
<td>0.57</td>
<td>1.15</td>
<td>0.41</td>
<td>2.20</td>
<td>0.0087</td>
<td>-0.011</td>
<td>-0.00009</td>
</tr>
<tr>
<td>$\sigma=-0.75$</td>
<td>0.65</td>
<td>0.006</td>
<td>0.70</td>
<td>0.45</td>
<td>0.61</td>
<td>1.25</td>
<td>0.42</td>
<td>2.25</td>
<td>0.0086</td>
<td>-0.011</td>
<td>-0.00009</td>
</tr>
<tr>
<td>$\sigma=-0.5$</td>
<td>0.65</td>
<td>0.013</td>
<td>0.70</td>
<td>0.45</td>
<td>0.60</td>
<td>1.21</td>
<td>0.42</td>
<td>2.30</td>
<td>0.0078</td>
<td>-0.009</td>
<td>-0.00012</td>
</tr>
<tr>
<td>$\sigma=-0.25$</td>
<td>0.65</td>
<td>0.028</td>
<td>0.70</td>
<td>0.45</td>
<td>0.61</td>
<td>1.14</td>
<td>0.43</td>
<td>2.39</td>
<td>0.0068</td>
<td>-0.008</td>
<td>-0.00016</td>
</tr>
<tr>
<td>$\sigma=-0.01$</td>
<td>0.67</td>
<td>0.06</td>
<td>0.69</td>
<td>0.45</td>
<td>0.67</td>
<td>1.21</td>
<td>0.45</td>
<td>2.55</td>
<td>0.0061</td>
<td>-0.012</td>
<td>-0.00012</td>
</tr>
<tr>
<td>$\sigma=0.01$</td>
<td>0.66</td>
<td>0.062</td>
<td>0.70</td>
<td>0.45</td>
<td>0.66</td>
<td>1.10</td>
<td>0.45</td>
<td>2.55</td>
<td>0.0063</td>
<td>-0.012</td>
<td>-0.00011</td>
</tr>
</tbody>
</table>

There is a negative correlation between $\lambda$ – the weight of early childhood education in the human capital function – and $\sigma$. Moreover, $\lambda$ fluctuates considerably with $\sigma$. This makes sense because investment in formal education is more attractive when the elasticity of the substitution of human capital increases (when $\sigma$ increases) because the individual would rather save during early childhood and invest their savings in latter stages of education. However, remember that we have the same target for schooling. Thus, the weight of schooling, $1 - \lambda$, decreases as it compensates for the increase in the return of schooling; i.e., the weight of early childhood education increases.

In Table 7 we re-evaluate the results of Section 6 changing the value of the complementarity parameter. We analyze the impact on formal education and income in 2008 when we keep the value of some types of education at the 1961 level. When complementarity is high (low $\sigma$), early childhood education has a higher impact on formal education and income. For instance, when the complementarity parameter is equal to -1 and early childhood education is kept constant at the 1961 level, formal education and income are 36.42% and 44.6% lower than in the benchmark case, whereas for complementarity equal to 0.01, it is 4.99% and 13.25% lower. Although early childhood education has a lower impact on income than formal education when the complementarity parameter is higher than -0.5, the impact on income is still impressive. Indeed, for sigma equal to 0.01 the early childhood impact on income (13.25%) is almost three times higher than the impact of early childhood on formal education (4.99%).

In Table 8 we re-evaluate the results of the last section for different values of the complementarity parameter. Thus, we keep constant total government expenditure on education but increase by 10%, for all years, the share of total government expenditure on pre-primary education for each $\sigma$. When complementarity is higher, the effect of this reallocation of government expenditure is also higher. For the case where $\sigma > 0$ the impact of a reallocation in government spending is low, 0.1%. Therefore, these results are robust to several values of complementarity, and it strengthens the importance of early childhood education.
Table 7: Sensitivity - Contrafactual results

<table>
<thead>
<tr>
<th>σ</th>
<th>-1</th>
<th>-0.75</th>
<th>-0.5</th>
<th>-0.25</th>
<th>-0.01</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early childhood at 1961 level</td>
<td>36.42%</td>
<td>28.09%</td>
<td>19%</td>
<td>10.99%</td>
<td>5.47%</td>
<td>4.99%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>σ</th>
<th>-1</th>
<th>-0.75</th>
<th>-0.5</th>
<th>-0.25</th>
<th>-0.01</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early childhood at 1961 level</td>
<td>44.6%</td>
<td>37.31%</td>
<td>28.86%</td>
<td>20.78%</td>
<td>14.42%</td>
<td>13.52%</td>
</tr>
<tr>
<td>Formal education at 1961 level</td>
<td>27.73%</td>
<td>27.68%</td>
<td>27.77%</td>
<td>27.87%</td>
<td>27.66%</td>
<td>27.59%</td>
</tr>
</tbody>
</table>

Table 8: Sensitivity - Contrafactual of reallocation of government expenditure in education

<table>
<thead>
<tr>
<th>σ</th>
<th>-1</th>
<th>-0.75</th>
<th>-0.5</th>
<th>-0.25</th>
<th>-0.01</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average increase in income</td>
<td>0.56%</td>
<td>0.54%</td>
<td>0.36%</td>
<td>0.25%</td>
<td>0.11%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

10 Conclusion

This paper studies the evolution of the human capital of American workers between 1961 and 2008. The novelty of this work is to incorporate to an otherwise standard continuous time life cycle model results from the early childhood literature and a human capital formulation close to that of Heckman and Cunha (2007) and Cunha et al. (2010). In this sense, early and formal education are complementary. We use the model to measure the evolution of early childhood human capital and its importance to labor productivity and formal education.

Early childhood skill formation is shown to be critical to understanding the evolution of U.S. income because the dynamic effect of the complementarity in human capital formation leads early childhood education to be used as input to formal education, which improves the return to the latter. Indeed, early childhood education explains 60.88% of the observed increase in the years of schooling in the U.S. Moreover, its impact on output is sizable and in the same order of magnitude as schooling. These results are shown to be very robust and reinforce, in a general equilibrium set–up, the results from the labor literature finding that the first stage of human capital formation is key for future income and cognitive development of individuals and, hence, nations.

A modified version of the model is used to analyze the role of government expenditures in education. The main result is that the government could have increased income per capita in the past by simply reallocating expenses in favor of early childhood education. Therefore, our results call for a reassessment of educational policies with increased emphasis on preschool education.

References


Head Start Bureau (2012). Head start program facts fiscal year 2012.


Appendix

A.1 The Evolution of Parental Childcare Time

Early childhood skill development is also highly influenced by parenting time, especially quality time they spend with the children, family income and neighborhood characteristics. There are not many time series information on these variables, however. One exception is Cardia and Gomme (2018) that estimates parameters of a childcare production function using data on primary and secondary childcare from the American Time Use Survey. They show that between 1965 and the average of the years 2003-2015 married woman in their prime childcare years (aged 24-29) more than doubled their time working in the market, while in the same period women aged 42-47 increased by only 35% their working time. In contrast, the first group decreased their primary childcare time (e.g., direct and exclusive activities) in the period, but the opposite happened for older woman. Along their entire lifetime the primary time mothers spend with their children increased, but not by much.

There are very strong evidence that maternal labor supply has a negative impact on the skill formation of their children, as for instance, Carneiro and Rodrigues (2009) and Agostinelli and Sorrenti (2018). The latter article also shows that childhood family income matters - increasing by thousand dollars the family income improves cognitive development by 4.4 percent of a standard deviation - but this effect is dominated by parental quality time. Using large longitudinal survey data from the UK Millennium Cohort Study, Bono et al. (2016) estimates that maternal time is a quantitatively important determinant of skill formation and that its effect declines with child age. The article finds evidence of long-term effects of early maternal time inputs on cognitive skill development. The Perry Program, and many similar programs of early intervention, include visits to the parents’ houses as a form of increasing their participation in the education of their children. The evidence, hence, on parenting time and family environment having positive impact on the skills of their children is solid. It is not clear, however, how these factors have evolved, or even improved, in the recent times. In the introduction and parts of the article, we argued that the variable $x$ is a reduced form variable that includes or represents factors affecting early childhood human capital. In this section, however, we add a separated and stylized variable representing effective parenting time (a proxy for quality and total parenting time) to the human capital function. Our objective is to evaluate how the addition of this variable changes the main results of Sections 5 and 6.

We now assume that, in addition to educational goods and services ($x$), early childhood human capital also depends on a variable $Q_x$ that we call parenting quality time or effective time. The new human capital function is given by:

$$h(T_s,x) = \left(\lambda (\alpha \sigma Q_x^{1-\alpha}) T_i \right)^\sigma + (1-\lambda) T_s^{\sigma} \right)^{\frac{1}{\sigma}} \tag{26}$$

Note that we are assuming an asymmetry with respect to parenting quality time, as it does not affect the second stage of human capital formation. Although it is true that parenting time is more important in early stages of child development, this is clearly a simplification. However, we want to stress the direct impact of parental time on early childhood skills. In the formulation above $1-\alpha_x$ is the elasticity of human capital with respect to effective time.

---

29 Other relevant references are Bernal and Keane (2010), Hsin and Felfe (2014) and Carneiro et al. (2015).

30 We simulated the model with several variations of this function (e.g., with $Q_x$ affecting both stages of skill formation) and results are qualitatively similar although, as expected, there are quantitative differences.
In the simulations, we assumed that $Q_x$ is exogenous and its value doubled from 1961 to 2008. This is of course an exaggeration, as the job market participation of woman, and especially mothers, increased sharply in the period. Although Cardia and Gomme (2018) documents that primary childcare time of mothers augmented from 1965 to 2000s for certain age groups, that is far from general and, in any case, the increase was not even close to 100% when it occurred. Hence, we think that the proposed variation of $Q_x$ is safely above reasonable estimates.

To our knowledge, there are no estimates of functional forms close to the human capital function above. Hence, we simulate the model for different values of $\alpha_x$, more precisely for $\alpha_x \in \{0.5, 0.7, 0.9, 1\}$. The smaller the alpha, the larger the weight of parental time in the skill development of younger children, and the case in which $\alpha_x = 1$ corresponds to the original model.

Table 9 presents the simulation results of the new model. The first column displays the values of the parameter $\alpha_x$ that are used in each model simulation. Columns 2 and 3 show how much $x$ and early childhood human capital, respectively, increased between 1961 and 2008. The last three columns present the contribution of early childhood education ($x$) in explaining the evolution of years of schooling, human capital and income per capita, respectively.

<table>
<thead>
<tr>
<th>$\alpha_x$</th>
<th>Evolution of Early Childhood Education ($x$) (times)</th>
<th>Evolution of Early Childhood Human Capital ($x^{\alpha_x}Q_x^{1-\alpha_x}$) (times)</th>
<th>Years of Schooling (%)</th>
<th>Human Capital (%)</th>
<th>Income per capita (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>6.117</td>
<td>3.5</td>
<td>27.1</td>
<td>39.96</td>
<td>14.14</td>
</tr>
<tr>
<td>0.7</td>
<td>6.124</td>
<td>4.38</td>
<td>40.69</td>
<td>56.21</td>
<td>24.65</td>
</tr>
<tr>
<td>0.9</td>
<td>6.294</td>
<td>5.61</td>
<td>54.24</td>
<td>70.67</td>
<td>37.39</td>
</tr>
<tr>
<td>1 (baseline model)</td>
<td>6.46</td>
<td>6.46</td>
<td>61</td>
<td>77.09</td>
<td>44.42</td>
</tr>
</tbody>
</table>

Table 9: Sensibility analysis - The evolution of parental time that was spent with child ($Q_x = 1 : 2$)

According to the results above, the variable $x$ increases by a magnitude similar to that observed in the original model, around 6 times for all $\alpha_x$ values. Hence, in this dimension quality time does not matter. However, the variation of early childhood human capital in the period falls when $\alpha_x$ falls. For instance, in the original model ($\alpha_x = 1$) early childhood human capital increases six times. In the case of $\alpha_x = 0.5$, it increases by only 3.5 times. This is so because the weight of $Q_x$, that grows by "only" 100% in the period, increases as $\alpha_x$ falls, while that of $x$, that grows more than six times in the period, increases with $\alpha_x$.

As one could expect, for values of $\alpha_x$ close to one, the benchmark calibration, results do not change significantly. For instance, for $\alpha_x = 0.9$, the contribution of $x$ to the observed evolution of formal education is now of 54% in the 1961-2008 period, instead of 61% as in the benchmark model. The figures for human capital are similar, 71% and 77%, respectively. In contrast, for smaller values of $\alpha_x$ the contribution of $x$ is less important. In these cases, the weight of this variable is smaller and the contribution of the expansion of quality time (assumed to grow by 100% in the period) larger. For $\alpha_x = 0.5$, the contribution of $x$ to the observed expansion of schooling falls to less than half the original figures, and the contribution to human capital growth is only 52% of that in the benchmark model.
There are two reasons that the latter case tends to be unrealistic. First, reduced form estimates of the impact of parental time on children skills are, in general, small. In Carneiro and Rodrigues (2009), for instance, it was estimated to be a small fraction of the standard deviation, which, in its turn, was not large. Hence, although there is not a direct correspondence between $\alpha_x$ and estimates in the literature, if anything one should expect small values of this variable. Second, we are assuming a very large and most probably unreasonable variation of quality time in the period, and this tend to reduce the contribution of $x$ to schooling and human capital. For realistic values of $\alpha_x$ and of $Q_3$ growth, results are closer to the original model.

A.2. TFP of Early Childhood Sector ($A_2$) Varying in Time

In the model calibration we assumed that the total factor productivity of the preschool education sector was equal to one in every year. In this section, we relax this assumption by assuming that $A_2$ equals one only in 2008 and we then recalibrate the model by allowing the growth rate of $A_2$ to be different from one over the period, and estimated endogenously.

Results are not too different from before, and the model matches very closely the data. In fact, the average percentage error of the targets is 0.37%, which is 0.11% closer to the data than the original model. This was expected since the model has now an additional parameter, $A_{2g}$, for the same number of targets. In addition, the estimated parameters do not vary much compared to the original model. The main different relates to the trend of the TFP growth rate of the formal education sector, that changed considerably because it is not evolving any longer relative to a constant $A_2$, but to one that also evolves over time.

Figure 7 shows the evolution of formal and preschool education prices. Both prices follow the same trends observed in the original model. In fact, the price of preschool and formal education increased 35.3% and 39.15%, respectively, corroborating the results of the benchmark model. Similarly, from Figure 8 we find, once again, a vigorous growth of preschool education in the period.

Figure 7: $A_2$ Varying in Time - Educational Prices

$A_{2g}$ explains little of the evolution of GDP and schooling. Figure 9 presents a counterfactual exercise in which the TFP of the preschool education sector is kept constant at the 1961 level. According to the simulations, GDP and years of schooling would be approximately only 1% and 0.6% lower in 2008 if the TFP of the preschool education sector was equal to its 1961 level.
Figure 8: Changing $A_2$ - Early Childhood Education

Figure 9: Constant level at 1961 for TFP of early childhood sector for a model with $A_2$ varying in Time - 1961 to 2008

(a) Years of Schooling

(b) GDP per capita

A.3 Additional Tables

Table 10: Endogenous calibration - Different informational set for targets

<table>
<thead>
<tr>
<th>Informational set</th>
<th>$\beta$</th>
<th>$\lambda$</th>
<th>$\tau_L^0$</th>
<th>$\tau_K^0$</th>
<th>$\tau_H^0$</th>
<th>$\tau_L^1$</th>
<th>$\tau_H^1$</th>
<th>$A_1^0$</th>
<th>$A_1^1$</th>
<th>$A_1^s$</th>
<th>$A_1^g$</th>
<th>$\tau_L^1$</th>
<th>$\tau_H^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$IS_1$</td>
<td>0.65</td>
<td>0.013</td>
<td>0.70</td>
<td>0.45</td>
<td>0.60</td>
<td>1.19</td>
<td>0.42</td>
<td>2.30</td>
<td>-0.0072</td>
<td>-9.5E-05</td>
<td>-0.019</td>
<td>-0.064</td>
<td>-0.019</td>
</tr>
<tr>
<td>$IS_2$</td>
<td>0.65</td>
<td>0.013</td>
<td>0.70</td>
<td>0.45</td>
<td>0.60</td>
<td>1.21</td>
<td>0.43</td>
<td>2.30</td>
<td>-0.0065</td>
<td>-3.1E-05</td>
<td>-0.022</td>
<td>-0.055</td>
<td>-0.022</td>
</tr>
<tr>
<td>$IS_3$</td>
<td>0.65</td>
<td>0.013</td>
<td>0.70</td>
<td>0.45</td>
<td>0.60</td>
<td>1.21</td>
<td>0.42</td>
<td>2.30</td>
<td>-0.0089</td>
<td>-1.2E-04</td>
<td>-0.024</td>
<td>-0.056</td>
<td>-0.024</td>
</tr>
</tbody>
</table>
### Table 11: Endogenous parameters - Government expenditure as percentage of GDP

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>U.S. data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leisure preference</td>
<td>β</td>
<td>0.14</td>
<td>Mean of average hours worked to sample</td>
<td>0.3171</td>
</tr>
<tr>
<td>Weight of early childhood in Human Capital Technology</td>
<td>λ</td>
<td>0.23</td>
<td>Mean of schooling to sample</td>
<td>11.7</td>
</tr>
<tr>
<td>Initial Labor Tax</td>
<td>τₜₜ</td>
<td>0.93</td>
<td>Average hours worked in 2008</td>
<td>0.307</td>
</tr>
<tr>
<td>Capital Tax</td>
<td>τₜₜ</td>
<td>0.41</td>
<td>Investment share in 2008</td>
<td>0.22</td>
</tr>
<tr>
<td>Initial Schooling Tax</td>
<td>τₜₜ</td>
<td>-0.85</td>
<td>Years of schooling in 2008</td>
<td>13.3</td>
</tr>
<tr>
<td>Initial Early Childhood Education Tax</td>
<td>τₜₜ</td>
<td>0.24</td>
<td>Worker share in sector 2 in 2008</td>
<td>0.0034</td>
</tr>
<tr>
<td>TFP initial goods sector</td>
<td>Aₜₕ</td>
<td>0.33</td>
<td>GDP per capita in 2008</td>
<td>1.028</td>
</tr>
<tr>
<td>TFP initial schooling sector</td>
<td>Aₜₕ</td>
<td>1.61</td>
<td>Worker share in sector 3 in 2008</td>
<td>0.0378</td>
</tr>
<tr>
<td>Trend of TFP goods sector</td>
<td>Aₜₜ</td>
<td>0.0016</td>
<td>Variance GDP per capita to sample</td>
<td>0.064</td>
</tr>
<tr>
<td>Trend of TFP schooling sector</td>
<td>Aₜₜ</td>
<td>0.0007</td>
<td>Mean GDP per capita growth rate between 1971 to 2008</td>
<td>0.0225</td>
</tr>
<tr>
<td>Trend of labor tax</td>
<td>τₜₜ</td>
<td>-0.0001</td>
<td>Variance of average hours worked to sample</td>
<td>0.0001</td>
</tr>
<tr>
<td>Trend of schooling tax</td>
<td>τₜₜ</td>
<td>-0.04</td>
<td>Variance of schooling to sample</td>
<td>2.478</td>
</tr>
<tr>
<td>Trend of early childhood education tax</td>
<td>τₜₜ</td>
<td>-0.1</td>
<td>Mean GDP per capita to sample</td>
<td>0.676</td>
</tr>
</tbody>
</table>

*Note: This sample is given by \{1961, 1971, 1980, 1990, 2000, 2008\}.*