Testing Production Functions Used in Empirical Growth Studies*

Pedro Cavalcanti Ferreira† João Victor Issler

Samuel de Abreu Pessôa

Graduate School of Economics - EPGE

Fundação Getulio Vargas

Abstract

We estimate and test alternative functional forms, which have been used in the growth literature, representing the aggregate production function for a panel of countries. Functional forms are confronted using a Box-Cox test, and results favor the mincerian formulation of schooling-returns to skills.

Keywords: Production Function; Growth; Box-Cox Test;

JEL classification codes: O47; C23

*We gratefully acknowledge the comments and suggestions of an anonymous referee, Costas Azariadis, Gary Hansen, Marcos B. Lisboa, Carlos Martins-Filho, Naércio Menezes, Alberto Trejos, Farshid Vahid, Martin Uribe, and several seminar participants around the world. All remaining errors are ours. Rafael Martins de Souza provided excellent research assistance. The authors also acknowledge the financial support of CNPq-Brazil and PRONEX.

†Corresponding author. Praia de Botafogo 190, 1125, Rio de Janeiro, RJ, 22253-900, Brazil. Email addresses of the authors are, respectively, ferreira@fgv.br, jissler@fgv.br, pessoa@fgv.br

1 Introduction

In this paper we estimate and test two alternative functional forms, that have been used in the growth literature, representing the aggregate production function for a panel of countries. The first model has a long tradition in this literature and was proposed by Mankiw Romer and Weil (1992), among others. The second is a mincerian formulation of schooling-returns to skills, traditionally used in the labor-economics literature, e.g., Mincer (1974), but recently incorporated into the growth literature as well; see Klenow, Rodriguez-Clare (1997), Hall and Jones (1999), and Bils and Klenow (2000). Islam(1995) recognizes that human capital is important in the growth process, but claims that the unresolved question is: "in what exact way"? Since the basic difference between these two competing models is the way in which human capital affects output – in the mincerian model human capital enters the production function exponentially while in the Mankiw, Romer and Weil model it enters the production function in levels – we propose here to distinguish between them by means of an econometric test based on the Box and Cox(1962) transformation.

Contrary to the bulk of the previous empirical growth literature, we estimate directly the logarithmic version of the production function, which makes performing a Box-Cox test a direct exercise. Panel-data estimation is performed using instrumental-variable techniques, after a careful search for appropriate instruments. Also, a variety of specification tests are performed to validate the final choice of model. The data used in our study are extracted from Summers and Heston (1991, mark 5.6) and Barro and Lee (1996), and includes 95 countries in different stages of economic development, with observations ranging from 1960 to 1985.

2 Model Specification

The Mankiw, Romer and Weil model (MRW),[1] has the following specification when constant returns of scale are not imposed:

$$Y_{it} = A_{it} K_{it}^{\alpha} H_{it}^{\phi} (L_{it} \exp(g \cdot t))^{\beta}, \tag{1}$$

where Y_{it}, K_{it}, H_{it}, L_{it}, and A_{it} are respectively output, physical capital, human capital, raw labor inputs, and productivity for country i in period t, where $i = 1, \cdots, N$, and $t = 1, \cdots, T$ and g is the exogenous technological progress, which is the same across countries. In per-worker terms, the equation above reduces to:

$$\ln y_{it} = \ln A_i + \alpha \ln k_{it} + \phi \ln h_{it} + (\alpha + \beta + \phi - 1) \ln L_{it} + \beta g \cdot t + \eta_{it}, \tag{2}$$

where lower-case variables are in per-worker terms, and A_{it} is decomposed into a time-invariant component A_i and an error component that varies across i and t, η_{it}.

In the mincerian specification (Mincer (1974)) there is only one type of labor in the economy, which has skill-level determined by educational attainment. It is assumed that the skill-level of a worker with h years of schooling is $\exp(\phi h)$ greater than that of a worker with no education at all, leading to the following function:

$$Y_{it} = A_{it} K_{it}^{\alpha} \left(\exp(\phi h_{it}) L_{it} \exp(g \cdot t)\right)^{\beta}. \tag{3}$$

In per-worker terms, the equation above reduces to:

$$\ln y_{it} = \ln A_i + \alpha \ln k_{it} + \beta \phi h_{it} + (\alpha + \beta - 1) \ln L_{it} + \beta g \cdot t + \eta_{it}. \tag{4}$$

Econometrically, the basic difference between equations (2) and (4) is whether human capital enters the (logarithmic version of the) production function in levels or in *logs*. We therefore estimate a general regression model:

$$\ln y_{it} = \ln A_i + \lambda_1 \ln k_{it} + \lambda_2 \left(\frac{h_{it}^{\theta} - 1}{\theta}\right) + \lambda_3 \cdot t + \lambda_4 \ln L_{it} + \eta_{it}, \tag{5}$$

which nests (2) and (4) even when constant returns to scale do not hold.

[1] Recently revisited by Bernanke and Gürkynak (2001).

3 Econometric Estimation, Testing, and Results

Consider the Box and Cox (1962) transformation for a given regressor:

$$\frac{x_{it}^{\theta}-1}{\theta}. \tag{6}$$

Since $\lim_{\theta\to 0}\frac{x_{it}^{\theta}-1}{\theta}=\ln(x_{it})$, and $\lim_{\theta\to 1}\frac{x_{it}^{\theta}-1}{\theta}=x_{it}-1$, for the logarithmic transformation to be valid we must have $\theta=0$, and for the series x_{it} to enter the regression in levels we must have $\theta=1$. The hypotheses that human capital enters the production function in levels or in *logs* can thus be tested using a Box-Cox transformation for the human-capital stock by means of a Wald test, with $H_0:\theta=1$ and $H_0:\theta=0$ respectively.

In estimating either (2) or (4) we must take into account that $\ln k_{it}$, $\ln h_{it}$, and h_{it} are correlated with η_{it}, which requires using instrumental-variable techniques. We propose using the following instruments for $\ln k_{it}$, $\ln h_{it}$, and h_{it}, respectively:

$$\frac{1}{N^i}\sum_{j\in\{N^i\}}\ln k_{jt-1}, \tag{7}$$

$$\frac{1}{N^i}\sum_{j\in\{N^i\}}\ln h_{jt-1}, \tag{8}$$

$$\frac{1}{N^i}\sum_{j\in\{N^i\}}h_{jt-1}, \tag{9}$$

where N^i represents the number of additional countries in the same continent that country i is in, and $\{N^i\}$ represents the set of countries in that continent that are not country i, i.e., (7), (8), and (9) represent, respectively, rest-of-the-continent average lagged (log of the) capital stock, (log of the) human capital stock, and level of the human capital stock. Hence, instruments are country specific.

Rest-of-continent instruments such as (7), (8), and (9) are promising, since one could expect *a priori* that countries in the same continent are similar in several dimensions and are relatively more integrated, making instruments correlated with regressors. The fact that country i is excluded from computing rest-of-continent averages makes the latter more likely to be uncorrelated with country's i error: η_{it}, which can be formally tested for over-identified models; a basic references is Sargan (1958).

With appropriate instruments we estimate (5)[2]. We assume that the variance of η_{it} does not change across t, although it is allowed to change across i. Due to this structure, our estimation method weights data in each equation by the reciprocals of the standard deviation of country-specific errors, similar to the procedure in weighted two-stage least squares. The only difference here to weighted two-stage least squares is that we use equation-specific instruments and not the whole set of instruments. The constant term in each regression equation uses country-specific dummies, similar to fixed-effect estimation in a completely linear setup.

Based on the results of Wald tests with $H_0 : \theta = 1$ and $H_0 : \theta = 0$, we could either choose one of the models (2) or (4), or reject (not reject) both. If one is chosen we then perform estimation of the chosen one using the same procedure described above: weighted instrumental-variable technique. Orthogonality tests between errors and instruments using the procedure in Sargan(1958) is performed. Since we have country-specific instruments we perform Sargan tests equation-by-equation.

The panel data set used ranges from 1960 to 1985, and combines macroeconomic data for 95 countries in the mark 5.6 of the Summers and Heston (SH, from now on) data set with human-capital measures extracted from Barro and Lee (1996). We decided to interpolate human-capital measures to fit annual frequency.[3] The time span was restricted from 1960 to 1985. The specific series used are the following: y_{it} is the ratio of real GDP (at 1985 international prices) and the number of workers in the labor force, extracted from SH; k_{it} is the physical capital series per worker. The physical capital series is constructed using real investment data from SH (at 1985 international prices) and the Perpetual Inventory Method;[4] h_{it} is Barro and Lee's

[2] In (5) we also need instruments for $\ln L_{it}$. We have used $\ln L_{it-1}$.

[3] Although this induces measurement error in human capital, the problem is relatively small, since human capital changes with a highly predictable pattern and the estimation technique used allows for regressors that are measured with error. To check the robustness of estimation results, we compared those using data with five-year intervals and those using yearly data. They were very close indeed.

[4] As for the initial capital stock, we followed Hall and Jones (1999) among many and approximated it by $K_0 = I_0/(g_I + \delta)$, where K_0 is the initial capital stock, I_0 is the initial investment expenditure, g_I is the growth rate of investment, and δ is the depreciation rate of the capital stock. The latter was set equal to 9%, but results barely changed when we used different values

(1996) series of average years of completed education of the labor force, interpolated (in levels) to fit annual frequency.

3.1 Model Estimation Results

Estimates of parameters in (5) are presented in Table 1 below.

< *include Table 1 here*>

First, the estimate of θ is 0.86, closer to unity, favoring the mincerian specification for the production function. Indeed, Wald test results for $\theta = 1$ and $\theta = 0$ did not reject the mincerian model, although strongly rejected the MRW model. Hence our final conclusion is in favor of the mincerian model against the MRW model. This is a relevant result, since a group of authors in the growth literature have recently made the case for the use of the mincerian form of the production function; see Hall and Jones (1999), Klenow and Rodriguez-Clare (1997) and Bils and Klenow (1999), among others. Because their argument is entirely based on microeconomic evidence – e.g., Psacharopoulos (1994) – the evidence presented here using the Box-Cox transformation confirms the appropriateness of their approach from a formal econometric point of view, using a macroeconomic model. At the same time, our evidence points toward the rejection of a competing alternative model also used extensively in the growth literature.

Second, despite the fact that Hall and Jones, Klenow and Rodriguez-Clare, and Bils and Klenow impose constant returns in production, a formal test rejected this hypothesis, since λ_4 is statistically significant[5]. To check whether or not our model choice depends on ruling out constant returns, we have performed Box-Cox testing assuming constant returns beforehand. The final choice of model is unaltered: we still do not reject the mincerian model although we do reject the MRW model.

In Table 2 we present the estimates of parameters of the mincerian model, i.e., equation (4).

< *include Table 2 here*>

for δ.

[5] See also the evidence agaisnt constant returns in Duffy and Papageorgiou (2000).

The reported estimates for α, ϕ, β, and g in Table 2 are close to what could be expected *a priori*: several calibrated studies use a capital elasticity $\alpha = 1/3$ (see Cooley and Prescott (1995)). Estimates in Gollin (2002) are close to 0.43 for a variety of countries. As discussed above, ϕ can be interpreted as a measure of the percentage increase in income of an additional year of schooling. Mincerian regressions usually find $\widehat{\phi} \simeq 10\%$ (Mincer (1974)). Moreover, Psacharopolos (1994), who surveys schooling-return evidence using a large set of countries, finds an average of 6.8% for OECD countries and 10.1% for the world as a whole – very close to our estimate. An average growth rate of productivity of about 2.2% a year is in line with the evidence on long-run growth presented by Maddison (1995). Last, we perform a Wald test for $\ln(A_i) = \ln(A), \forall i$, when estimating (4). Results overwhelmingly reject these restrictions (a p-value of 0.000), showing that productivity indeed varies across countries.

Because we want to check the specification of our structural model, we performed a series of Sargan (1958) tests (equation-by-equation). If we take the significance level to be 5%, from a total of 95 country-regressions, 15 countries rejected the null in this test. This is about 16% of the sample of countries, a relatively low number. Although in terms of number of countries these rejections are relatively small, since the data for each country are weighted by the reciprocal of the standard deviation of its error term in computing estimates, it could happen that including these countries makes a big difference in terms of parameter estimates. To check if this was a potential problem, we ran mincerian regressions excluding from our sample of countries those for which we rejected orthogonality at the 5% level in the Sargan test. The results of this exercise showed overwhelmingly that estimates did not changed very much when these countries were excluded. For the restricted sample of countries, parameter estimates are the following: $\widehat{\alpha} = 0.42$, $\widehat{\beta} = 0.39$, $\widehat{\phi} = 0.089$, and $\widehat{g} = 0.029$.

4 Conclusion

In this paper, we examined several features of the production functions used in the growth literature. The most relevant result is related to the form human capital should enter in the production function. Since the basic difference between the two most popular models used in the field – the mincerian model and the Mankiw, Romer and Weil model – is whether human capital affects output exponentially or in levels, we propose here to distinguish between them by means of an econometric test based on the Box and Cox(1962) transformation. Contrary to the bulk of the previous empirical growth literature, we estimate directly the logarithmic version of the production function, which makes performing a Box-Cox test a direct exercise.

The tests conducted here show unequivocally that the mincerian model is more appropriate than the augmented neoclassical growth model when we consider the way in which the production function should be written. This is a relevant result, since a group of authors in the growth literature have recently made the case for the use of the mincerian form of the production function; e.g., Hall and Jones (1999), Klenow, Rodriguez-Clare (1997) and Bils and Klenow (1999), among others. Because their argument is entirely based on microeconomic evidence, the results presented here validate their previous choice of production function from a formal econometric point of view. Moreover, our estimated coefficients are consistent with previous microeconomic evidence.

References

[1] **Barro, R. and J. W. Lee, 1996.** "International Measures of Schooling Years and Schooling Quality," *American Economic Review*, **86**(2): 218-223.

[2] **Bernanke, Ben S. and Gürkaynak, Refet S. 2001.** "Is Growth Exogenous? Taking Mankiw, Romer and Weil Seriously," NBER Macroeconomics Annual 2001, Ben S. Bernanke and Julio J. Rotemberg, Eds.Cambridge, MA: The MIT Press, 11-57.

[3] **Bils, M. and P. Klenow 2000.** "Does Schooling Causes Growth?," *American Economic Review*, **90**(5): 1160-1183.

[4] **Box, G. E. P. and Cox, D. R. 1962.** "An Analysis of Transformations," *Journal of the Royal Statistical Society*, Series B: 211-243.

[5] **Cooley, T. and Prescott, E., 1995.** "Economic Growth and Business Cycles," in Cooley, T. and Prescott, E. (eds.), *Frontiers of Business Cycle Research*, Princeton, Princeton University Press.

[6] **Duffy, J. and C. Papageorgiou, 2000.** "A Cross-Country Empirical Investigation of the Aggregate Production Function Specification," *Journal of Economic Growth*, **5**(1): 87-120.

[7] **Gollin, D., 2002.**"Getting Income Shares Right: Self Employment, Unincorporated Enterprise, and the Cobb-Douglas Hypothesis," *Journal of Political Economy*, **110**(2): 458-474.

[8] **Hall, R.E. and C. Jones, 1999.** "Why do some countries produce so much more output per worker than others?" *Quarterly Journal of Economics*, **114**(February): 83-116.

[9] **Islam, N., 1995.** "Growth Empirics: A Panel Data Approach," *Quarterly Journal of Economics*, **110**: 1127-1170.

[10] **Klenow, P. J. and A. Rodríguez-Clare, 1997.** "The neoclassical revival in growth economics: has it gone too far?" *NBER Macroeconomics Annual 1997*, Ben S. Bernanke and Julio J. Rotemberg, Eds. Cambridge, MA: The MIT Press, 73-103.

[11] **Mankiw, N. G., Romer, D. and D.Weil. 1992.** "A Contribution To The Empirics of Economic Growth." *The Quarterly Journal of Economics* **107**(May): 407-437.

[12] **Maddison, A., 1995.** "*Monitoring the World Economy: 1820-1992*," Paris: OECD.

[13] **Mincer, J., 1974.** *Schooling, Experience, and Earning,* National Bureau of Economic Research, distributed by Columbia University Press.

[14] **Psacharopoulos, G., 1994.** "Returns to Investment in Education: A Global Update," *World Development*, **22**(9): 1325-1343.

[15] **Sargan, J. D., 1958.** "The Estimation of Economic Relationships using Instrumental Variables," *Econometrica*, vol. **26**(3): 393-415.

[16] **Summers, R. and A. Heston, 1991**. "The Penn World Table: An Expanded Set of International Comparisons,1950-1988," *Quarterly Journal of Economics*, **106**(May): 327-368.

Table 1: Estimates of the Models in Box-Cox Form

Equation: $\ln y_{it} = \ln A_i + \lambda_1 \ln k_{it} + \lambda_2 \left(\frac{h_{it}^{\theta}-1}{\theta}\right) + \lambda_3 \cdot t + \lambda_4 \ln L_{it} + \eta_{it}$						
Estimated Parameters/(Std. Errors)					Box-Cox Wald Test	
λ_1	λ_2	λ_3	θ	λ_4	$\theta = 1$ (p-value)	$\theta = 0$ (p-value)
0.4285	0.0537	0.0098	0.8622	-0.1216	0.1657	0.0000
(0.0059)	(0.0102)	(0.0004)	(0.0994)	(0.0193)		

Table 2: Estimates of the Mincerian Growth Model (Log-Level Model)

Equation: $\ln y_{it} = \ln A_i + \alpha \ln k_{it} + \beta\phi h_{it} + \beta g \cdot t + (\alpha + \beta - 1) \ln L_{it} + \eta_{it}$,			
Estimated Parameters/(Std. Errors)			
α	ϕ	g	β
0.4306	0.0909	0.0221	0.4501
(0.0059)	(0.0090)	(0.0016)	(0.0200)