Interest Rates in Trade Credit Markets

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Abstract

Despite strong evidence that suppliers of inputs are informed lenders, the cost of trade credit typically does not vary with borrowing firm characteristics. We solve this puzzle by demonstrating that it is optimal for suppliers to keep the riskier firms indifferent between trade credit and loans from uninformed lenders. Because these uninformed loans vary across industries but not with firm characteristics, the same pattern applies to the cost of trade credit. The model predicts that the cost of trade credit is more likely to vary with firm characteristics in industries that are plagued by moral hazard problems or financial distress.

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1 Introduction

In the U.S., suppliers of inputs to production processes extend a significant amount of credit to their customers using standardized trade credit contracts.\(^1\) In these contracts, interest rates are determined by a discount for early payment. For instance, Petersen and Rajan (1994) and Ng, Smith and Smith (1999) show that a common trade credit contract combines a 30 day maturity with a two percent discount for early payment within 10 days of the invoice (2-10 net 30 loans). For all practical purposes, foregoing the deadline for the discount is equivalent to accepting a 20-day extension of credit at an annual interest rate of 44 percent. Suppliers, therefore, can align the cost of trade credit with the borrower’s risk by properly varying the terms of the discount. Indeed, these terms do vary across industries.

And yet, Giannetti, Burkart and Ellingsen (2008) cannot tie the discounts for early payment to the borrowers’ credit risk. This finding is hard to reconcile with the common view that suppliers are informed lenders, who, as Petersen and Rajan (1997) show, have a comparative advantage in identifying firms with growth potential.\(^2\) Presumably, the suppliers should use their informational advantage to increase the interest rates paid by the riskier firms.

We solve this puzzle by arguing that competition with uninformed lenders makes it difficult for informed suppliers to align the cost of trade credit with their customers’ risk. More specifically, any attempt to selectively raise the interest rates paid by the riskier firms will induce them to borrow from uninformed lenders, whose interest rates overestimate the odds that the debt contract will be honored. We demonstrate that it is optimal for suppliers to keep the riskier firms indifferent between trade credit and loans from uninformed lenders. Because the cost of these uninformed loans is likely to vary across industries but not with firm characteristics, the same pattern applies to the cost of trade credit.

To understand the main ideas of our paper, consider an industry with a continuum of firms, \(N\) suppliers of inputs and a competitive banking sector summarized by a representative

\(^1\)See Rajan and Zingales (1995) for the importance of trade credit in the G7 countries (Canada, France, Germany, Italy, Japan, U.K., and the U.S.).

\(^2\)A vast literature in corporate finance claims that the suppliers’ informational advantage explains why trade credit is pervasive. In particular, Smith (1987), Mian and Smith (1992) and Biais and Gollier (1997) argue that the sales effort of suppliers makes it easier for them to assess their customers’ credit risk.
bank. The firms seek financing to undertake a profitable project, whose possible outcomes are two: a positive return on the investment (success) or total loss (failure). While a bank loan is the standard source of external financing, some firms may have the option of using trade credit to finance the project.

An informational advantage explains why suppliers offer trade credit in our model. We assume that the probability that the project succeeds depends on firm-specific risk factors – the types – that are distributed in the positive interval \([\xi, 1]\). Without loss of generality, the probability of success, \(p^t\), increases with the firm’s type. When firms seek financing to the project, they know their own types and so do their main suppliers. The other suppliers and the bank, on the other hand, do not know the types.

Will the informed suppliers vary the cost of trade credit with the firm-specific risk factors? To answer this question, we build upon a key observation: private information gives market power to the informed suppliers. As suggested by standard monopoly pricing, a sufficiently inelastic demand for inputs (and, by extension, for credit) makes it optimal for the informed suppliers to raise the cost of trade credit until it reaches the cost of the borrower’s outside option, which, in our model, is the interest rate of a bank loan.\(^3\) Since the bank cannot vary the cost of its loans with information that is privy to the informed suppliers, the latter cannot use their information either, if they match the cost of trade credit to the bank rate. Accordingly, we demonstrate that, in equilibrium, the cost of trade credit does not vary with firm characteristics, if the elasticity of the demand for inputs is below a certain threshold \(\bar{\epsilon}\).

If the elasticity of demand is larger than \(\bar{\epsilon}\), then it is optimal for the informed suppliers to lower the cost of trade credit for the safer firms in order to boost their demand for trade credit. Firms that have access to trade credit are therefore split into two groups: The riskier ones get trade credit offers whose cost matches the interest rate of a bank loan, and the safer firms pay lower interest rates that decrease with their probability of success.

This implication of our model is interesting, for two reasons. First, there is evidence that suppliers do not treat all customers alike when they negotiate trade credit contracts. For instance, Ng, Smith and Smith (1999) show that suppliers occasionally waive penalties for

\(^3\)Brennan, Maksimovic and Zechner (1988) also argue that the elasticity of the demand for inputs determines the cost of trade credit. In their model, the supplier is a monopolist in the product market.
late payments, which is equivalent to selectively reducing the cost of trade credit. Second, and more importantly, testable models of interest rates in trade credit markets should predict when or where the cost of trade credit is more likely to vary with firm characteristics. As we argue below, three predictions of this sort arise from the equilibrium that allows for trade credit contracts to vary with firm characteristics.

A standard tradeoff, between margin of profit and volume of trade, determines the optimal interest rates in the trade credit markets. On the one hand, raising the interest rates increases financial profits per unit of profits. On the other hand, it decreases the demand for trade credit. All things equal, suppliers have more to gain from trade credit if their cost of raising funds is low. Accordingly, these suppliers have stronger incentives to protect the volume of trade credit for their most valuable customers, namely, the safer firms. A lower cost of funds, therefore, makes it more likely that the cost of trade credit varies with firm characteristics.

While a low cost of funds provides incentives for suppliers to reduce the cost of trade credit for the safer firms, financial distress reduces the expected debt repayment, thereby inducing the uninformed bank to increase interest rates. The higher bank rates let the informed suppliers selectively raise the cost of trade credit for the riskier firms while protecting the volume of trade credit for the safer firms. Our model, therefore, predicts that the cost of trade credit is more likely to vary with firm characteristics in financially-distressed industries.

A similar argument implies that the cost of trade credit is more likely to vary with firm characteristics in industries plagued by moral hazard. As Burkart and Ellingsen (2004) point out, suppliers can mitigate moral hazard problems more efficiently than banks, because trade credit is extended in kind rather than cash. It then follows that moral hazard problems weaken the banking sector’s ability to compete with trade credit, increasing the margins of profits of the informed suppliers and, consequently, strengthening their incentives to vary interest rates with the borrower’s risk. Hence, moral hazard doesn’t explain the existing evidence on the cost of trade credit, although it may explain why trade credit is pervasive.

This paper builds primarily on Biais and Gollier (1997) and Burkart and Ellingsen (2004). In Biais and Gollier, suppliers can identify firms whose credit risk is overestimated by banks. Knowing that these firms’ credit lines are unduly low, suppliers are willing to fill their financing
needs. Burkart and Ellingsen’s paper argues that loans in kind (as opposed to cash) are less vulnerable to moral hazard problems. As such, suppliers may extend credit to firms that have exhausted their ability to borrow from banks. In Biais and Gollier as well as Burkart and Ellingsen’s model, the cost of trade credit would vary with the suppliers’ private information, were the customers to have different levels of credit risk.

The remainder of the paper is organized as follows. Section 2 describes the basic model. Section 3 shows how firm-specific risk factors and industry characteristics determine the cost of trade credit. Section 4 introduces moral hazard problems in the model and shows that they make the cost of trade credit more sensitive to firm characteristics. Section 5 discusses the robustness of the results to different information structures and richer trade credit contracts. Section 6 concludes. Proofs that are not in the text can be found in the appendix.

2 The Model

2.1 Sequence of events and information structure

Consider a risk-neutral economy with a risk-free rate \( r \), \( N \) suppliers of inputs, a competitive banking sector summarized by a representative bank, and a continuum of firms \( C \) with types in the interval \([ t, 1]\), where \( t > 0 \). In this economy, the firms seek financing to undertake a project, whose possible outcomes are two: a positive return on the investment (success) or total loss (failure). When we add firm-specific factors to those that are intrinsic to the project, the probability that the project succeeds, \( p^i \), increases with the firm’s type: \( p^i = tp \), with \( p \in (0, 1) \) and \( t \in [t, 1] \).

While a bank loan is the standard source of financing, suppliers may finance the project by extending trade credit. Neither the bank nor the suppliers observe the project’s return without a verification cost. As in Townsend (1979) and Gale and Hellwig (1985), verification costs imply that outside equity isn’t an optimal financing contract. Hence, firms rely on debt-like instruments to finance the project, whether the lender is the bank or a supplier.\(^4\)

\(^4\)In our model, verification costs focus the financing decisions on the debt-like instruments that prevail in the trade credit markets. In a multi-period setting, Fluck (1998) demonstrates that equity financing may be optimal, even if cash flows are not verifiable.
As figure 1 shows, the game begins at date 0, when the firms seek financing to purchase inputs for the project. At this time, the firms know their own types, and so do their main suppliers. The bank and the other suppliers, on the other hand, know only that the types are distributed across $[t, 1]$ with a cumulative distribution $F_0(t)$ and density function $f_0(t)$. Intuitively, the informational advantage of the main suppliers is the outcome of unmodelled repeated purchases that give rise to most-favored business relationships. We model these special relationships by a set $\{T^n\}_{n=1}^N$, where $T^n$ is the set of firms whose main supplier is $n$. We assume that each firm has one (and only one) main supplier, that is, $T^l \cap T^m = \emptyset$ for any $l \neq m$ and $\bigcup_{n=1}^N T^n = \mathcal{C}$. To ensure that the suppliers are symmetric at $t = 0$, we also assume that, for any $n$, the types in $T^n$ belong to the interval $[t, 1]$ with the cumulative distribution $F_0(t)$. The distribution $F_0(t)$, the project-specific risk factor $p$ and the partition of firms $\{T^n\}_{n=1}^N$ are all common knowledge.

In addition to allowing a better assessment of the firms’ credit risk, special business relationships give the main suppliers a first-mover advantage. At date 1, they may bundle sales of inputs to trade credit through take-it-or-leave-it offers to their preferred customers.\(^5\) The main suppliers know, however, that if they do not reach an agreement with their preferred customers, then the latter may borrow from one of the uninformed lenders (the bank or the other suppliers) at date 2. After securing the funds, the firms buy the inputs and undertake

\(^5\)The main results of our paper hold under less extreme assumptions on how the main suppliers and their preferred customers reach an agreement over the terms of trade credit.
the project at date 2. The payoffs realize at date 3, when the firms repay the debt (if possible) and distribute the residual cash flow to shareholders.

2.2 Technology

In the U.S., trade credit contracts often let firms pay for the purchased goods after delivery, without charging a positive interest rate. Of course, trade credit contracts at zero interest rates are costly for the suppliers. For these contracts to make economic sense, they must benefit the suppliers in some other way.\(^6\)

Regardless of the reason for suppliers to subsidize trade credit, one would expect that the optimal subsidy varies with the borrower’s characteristics. One way for suppliers to vary the subsidy is to offer trade credit contracts with different discounts for early payment. Giannetti, Burkart and Ellingsen (2008) show, however, that the borrowers’ risk characteristics do not explain the terms of the discount for early payment.

To focus our analysis on the interest rate that is implicit in the discount for early payment, we assume that the market for inputs is competitive and endow the suppliers with a constant-return-to-scale technology. While competition rules out abnormal profits in the market for inputs, constant returns to scale imply that the equilibrium input price is equal to the marginal cost, which we normalize to one. If the input price is constant, then the cost of trade credit is the only instrument available for the suppliers to vary their pricing decisions with the borrowers’ characteristics. This is the worst-case scenario for a model that aims to explain why the cost of trade credit does not vary with firm characteristics.

Unlike the suppliers, the buyers of inputs can fetch abnormal profits in the product market. These firms are endowed with an investment opportunity whose possible outcomes are two: success or failure. In case of success, the return of investing \(I\) units of input in the project is \(Q(I)\). The production function \(Q(I)\) is increasing and strictly concave on the investment, satisfying \(Q(0) = 0\) and the standard Inada conditions.\(^7\) In contrast, failure destroys the investment, once we take into account the verification costs.

\(^6\)Daripa and Nilsen (2009), for instance, argue that subsidized interest rates induce cash-constrained firms to keep higher inventory levels, thereby increasing their suppliers’ profits in the market for inputs.

\(^7\)The Inada conditions are \(\lim_{I \to 0} Q'(I) = \infty\) and \(\lim_{I \to \infty} Q'(I) = 0\).
2.3 Trade credit contracts

Since competition drives the price of the input to its marginal cost, suppliers design trade credit contracts that maximize their expected financial profits. We shall restrict our attention to linear debt contracts, which are pervasive in the trade credit markets. Assuming linear contracts from the onset may be interpreted as a short-cut to focus the analysis on the most relevant determinants of interest rates in the trade credit markets.\(^8\)

To characterize the optimal trade credit contracts, we must take into account whether the borrowing firm is a preferred customer or not. Trade credit transactions with preferred customers are particularly appealing for the suppliers, for two reasons. First, they can vary the interest rate with the borrower’s risk. Second, they can make preemptive trade-credit offers. Of course, this first-mover advantage does not insulate the informed suppliers from competition in the credit markets. In particular, they cannot lend at an interest rate that is higher than the cost of the borrower’s alternative source of financing.

Leaving aside trade credit from a main supplier, what is the least costly financing alternative for the firms? As in Biais and Gollier (1997), we assume that the bank’s cost of funds is the riskless rate \(i\), while the cost of funds of the suppliers is \(i + c\), with \(c > 0\). Uninformed suppliers, therefore, cannot compete with the bank in the market for loans. A bank loan is the best alternative for a firm that does not agree with its main supplier with respect to the terms of trade credit. Note, however, that trade credit breaks down if the suppliers’ cost of funds is too high. To assure that trade credit arises as an equilibrium outcome, we assume:

**Assumption 1** The suppliers’ cost of funds satisfies \(1 + i + c < \frac{1+i}{\int_0^1 G(t)dt}\).

2.4 The borrowing firms’ outside option: A bank loan

Free entry drives the bank’s expected profit to zero. To see what type of restriction this zero-profit condition implies, consider a standard debt contract \((I, A(I))\), where \(I\) is the amount of the loan and \(A(I)\) is the promised repayment. When the bank offers the debt contract, it knows that the borrower will pay \(A(I)\) if and only if its investment in the project pays off.

\(^8\)Section 5 demonstrates that the main insights of our paper are robust to optimal nonlinear contracts, if default imposes small economic losses on suppliers.
From the bank’s viewpoint, the probability that the project succeeds is 
\[ p_{E_1[t]} = p \int_1^t t dF_1(t), \]
where \( F_1(t) \) is the updated distribution of the type of a firm that requests a bank loan. For the bank’s expected profit to be zero, its expected payment, \( p_{E_1[t]}A(I) \), must be equal to its cost of funds, \((1 + i)I\), implying that the interest rate of the debt contract is

\[ r^B \equiv \frac{A(I)}{I} - 1 = \frac{1 + i}{p_{E_1[t]}} - 1. \]  

Equation (1) shows that the interest rate \( r^B \) decreases with the expected type of the borrowing firm (i.e., \( E_1[t] \)). Competition in the banking sector, therefore, elicits incentives for loan contracts that screen the borrowers’ types. It is easy to show, nonetheless, that, in our model, the bank cannot screen the types, because any loan contract that is optimal for a type-\( t \) firm is also optimal for the other types. This feature of the model preserves the informed suppliers’ informational advantage, which is crucial to the purposes of our paper, while allowing for risk characteristics to vary across firms.

It then follows from Gale and Hellwig (1985) that a standard debt contract with a promised return \( r^B = \frac{1+i}{p_{E_1[t]}} - 1 \) is the least costly bank loan. In the optimal debt contract, the promised payment \( A(I) \) varies with the loan amount, but its promised return doesn’t. To characterize the optimal debt contract it thus suffices to pin down the loan amount that maximizes the value of a representative firm that borrows from the bank, that is, the investment \( I_{E_1[t]} \) that makes the marginal productivity of investment, \( p_{E_1[t]}Q'(I_{E_1[t]}) \), equal to its marginal cost, \( 1 + i \). The necessary and sufficient condition that characterizes the optimal investment is

\[ Q'(I_{E_1[t]}) = \frac{1 + i}{p_{E_1[t]}}. \]  

The optimal debt contract, therefore, lends \( I_{E_1[t]} \), in exchange for the borrower’s promise to pay \((1 + r^B)I_{E_1[t]} \) after the project’s payoff realizes.

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9More formally, consider any two standard debt contracts, \((I^k, A(I)^k)_{k \in \{T, \hat{T}\}} \), that finance \( I^k \) in exchange for a promised payment \( A(I)^k \). For the debt contracts to screen some of the firms, there must exist types \( t \in T \) and \( \hat{t} \in \hat{T} \) such that \( p'[Q(I^T) - A(I^T)] > p'[Q(I^{\hat{T}}) - A(I^{\hat{T}})] \) and \( p'[Q(I^T) - A(I^T)] \leq p'[Q(I^{\hat{T}}) - A(I^{\hat{T}})] \). These two conditions cannot hold simultaneously, though, because \( p'[Q(I^T) - A(I^T)] > p'[Q(I^{\hat{T}}) - A(I^{\hat{T}})] \) implies that \( p'[Q(I^T) - A(I^T)] > p'[Q(I^{t'}) - A(I^{t'})] \) for any \( t' \neq t \). Hence, if a loan contract is optimal for a type-\( t \) firm, then it is also optimal for the other types.
As it turns out, the optimal debt contract can be implemented by a linear contract that lets the borrowers pick the loan amount. To see this, consider the investment problem of a type-\( t \) firm that finances the inputs at an interest rate \( r \):

\[
\max_I p^t I\left[ Q(I) - (1 + r)I \right].
\]  

(3)

The objective function (3) takes into account that the firm benefits from the project if it succeeds. With probability \( 1 - p^t \), the firm gets into operational problems that destroy the project’s payoff. The first order condition of program (3), which is also sufficient, is

\[
Q'(I^*) = 1 + r.
\]  

(4)

Plugging \( r = \frac{1+i}{pE_1[t]} - 1 \) into the first order condition (4) yields \( Q'(I^*) = \frac{1+i}{pE_1[t]} \), which is the first order condition (2) for the value-maximizing investment of the representative type \( E_1[t] \). Hence, we can assume, without loss of generality, that a linear debt contract at the interest rate \( r^B = \frac{1+i}{pE_1[t]} - 1 \) is the firms’ alternative for borrowing from the informed supplier.

3 \hspace{1em} \textbf{Equilibrium in the trade credit market}

This section is divided in three parts. The first part derives the optimal trade credit contracts, taking as given the updated belief \( F_1(t) \) that determines the cost of borrowing from the bank. In the second part, we obtain \( F_1(t) \) as part of the equilibrium of the game. The last part of the section investigates the likelihood that the cost of trade credit varies with firm characteristics.

3.1 \hspace{1em} \textbf{The optimal trade credit contracts of informed suppliers}

The focus of this section is on trade credit transactions between a firm and its main supplier of inputs. Consistently with Petersen and Rajan (1997), we argue that an informational advantage shapes these trade credit transactions: The main supplier is the only lender that knows the types of its preferred customers. The main supplier of a type-\( t \) firm thus solves

\[
\max_{r'} \left( p^t (1 + r') - (1 + i + c) \right) I^*(r')
\]  

(5)

subject to \( \frac{1+i+c}{p^t} - 1 \leq r' \leq r^B \).

(6)
The objective function is the informed supplier’s expected profit of extending trade credit to a type-\(t\) firm at an interest rate \(r^t\). The loan amount, \(I^*(r^t)\), is the optimal purchase of inputs given the interest rate \(r^t\) (see the first order condition (4)). With probability \(p^t\), the borrower can pay principal plus interest, \((1+r^t)I^*(r^t)\), but, with probability \(1-p^t\), operational problems destroy the project’s payoff. Whether the project succeeds or not, the supplier bears the cost of producing the input plus its own cost of funds, that is, \((1+i+c)I^*(r^t)\).

When choosing the interest rate, the informed supplier takes into account that a necessary condition for lending to a type-\(t\) firm to be profitable is that \(r^t\) is larger than \(\frac{1+i+c}{tp^t} - 1\). The interest rate cannot be too high, though, or else the firm is better off borrowing from the bank at the interest rate \(r^B\). These two restrictions are summarized in (6).

The only reason for a solution to Program (5) not to exist is the upper bound on the interest rate (\(r^t \leq r^B\)). If \(r^B\) is lower than the break-even rate, then the program’s opportunity set is empty, meaning that the informed supplier will not offer trade credit to the type-\(t\) firm. Using \(p^t = tp\), a necessary and sufficient condition for trade credit to be profitable for the informed supplier is \(r^B \geq \frac{1+i+c}{tp} - 1\), or equivalently

\[
t \geq \frac{1 + i + c}{p(1 + r^B)}.
\]  

(7)

Our next task is to characterize the interest rate that solves the maximization problem (5), assuming that trade credit is profitable (i.e., \(t \geq \frac{1+i+c}{p(1+r^B)}\)). To do this, note first that \(r^t = \frac{1+i+c}{tp^t} - 1\) yields zero expected profits for the informed suppliers, while any slightly larger rate implies strictly positive expected profits. The optimal interest rate charged to a \(t\)-type, therefore, is either \(r^B\) or lies in the open interval \((\frac{1+i+c}{tp^t} - 1, r^B)\).

A standard trade-off, between margin of profit and volume of trade credit, determines the optimal interest rate. To rule out the uninteresting case that it is always optimal for the informed suppliers to raise the cost of credit as much as possible, we assume: \(^{10}\)

**Assumption 2** Let \(\epsilon(r) = \frac{-\frac{dI^*(r)}{dr}}{I^*(r)}\) be the interest-elasticity of the demand for inputs. Thus, \(\epsilon(r)\) is non-decreasing in \(r\).

Given Assumption 2, Proposition 1 characterizes the interest rate that solves Program 5.

\(^{10}\)Assumption 2 holds, for example, if \(Q(I) = I^\alpha\) with \(\alpha \in (0, 1)\).
Proposition 1  It is profitable for any informed supplier to offer trade credit if and only if the firm’s type is \( t \geq \frac{1+i+c}{p(1+r_B)} \). In this case, a necessary and sufficient condition for \( r^t \) to be the optimal interest rate is \( \epsilon(r^t) \leq \tau(r^t, t) \), where \( \tau(r^t, t) = \frac{tp(1+r^t)}{tp(1+r^t) - (1+i+c)} \). The optimal interest rate \( r^t \) strictly decreases with \( t \), for any \( r^t < r^B \). If \( r^t = r^B \), then the optimal interest rate does not vary with the borrower’s firm-specific characteristics.

Proposition 1 shows that the cost of trade credit falls with \( t \), unless the informed supplier raises the interest rate to the maximum level that firms are willing to pay, that is, the bank rate \( r^B \). From Proposition 1, the necessary and sufficient condition for \( r^t = r^B \) to be optimal is \( \epsilon(r^B) \leq \tau(r^B, t) \). Fixed \( r^B \), the cutoff \( \tau(r^B, t) \) decreases with the firm’s type, \( t \). Hence, \( \epsilon(r^B) \leq \tau(r^B, 1) \) ensures that the cost of trade credit never varies with firm characteristics.

As Petersen and Rajan (1997) show, the demand for trade credit in the U.S. seems to be fairly inelastic; a finding that Propositions 1 relates to interest rates that do not vary with firm characteristics. Note, however, that Proposition 1 does not provide a complete characterization of the optimal trade credit contracts. The cutoff type that has access to trade credit, \( \frac{1+i+c}{p(1+r_B)} \), and the optimal interest rate, \( r^t \), depend on the bank rate \( r^B \), which is an endogenous variable of our model. The next section integrates the trade credit market with the market for bank loans, thereby solving for the equilibrium of the game.

### 3.2 Characterization of the equilibrium

To describe the normal form of the game, consider the following pure strategies and belief:

\[
\mathcal{E} = \left\{ (I^B, r^B), (I^*(r^t), r^t) \right\}_{t \in [\underline{t}, 1]} \cup \left\{ 1_{\text{bank}}^{t \in [\underline{t}, 1]} ; F_1(t) \right\}. \tag{8}
\]

In the profile \( \mathcal{E} \), the bank’s strategy is a standard debt contract, \( (I^B, r^B) \), that is available to all firms. The debt contract lends \( I^B \), in exchange for a promised payment that implies an interest rate \( r^B \). Unlike the bank, the informed suppliers know the types of its customers. Their strategies, therefore, consist of a set of trade credit contracts, \( (I^*(r^t), r^t) \), that can be tailored to the borrower’s firm-specific risk factors. All trade credit contracts finance purchases of inputs that let the borrower implement the optimal scale of the project, \( I^*(r^t) \), given the interest rate \( r^t \). In addition to varying the interest rate with the borrower’s risk,
the informed suppliers can deny trade credit to types below a cutoff \( \bar{t} \in [t, 1] \). The firms’ strategies consist of the indicator functions \( \{1^t_{\text{bank}}\}_{t \in [t, 1]} \). For each \( t \), the indicator function takes value one if a bank loan maximizes the type-\( t \) firm’s expected profit and value zero if trade credit is the profit-maximizing loan. Finally, the cumulative distribution \( F_1(t) \) describes the bank’s belief on the type of a firm that asks for a bank loan.

The equilibrium concept we use is Perfect Bayesian Equilibrium (PBE). For \( \mathcal{E} \) to be a PBE, the strategies and belief must satisfy two conditions. First, each player’s strategy must maximize its expected payoff, conditioned on the other players’ strategies and the equilibrium belief \( F_1(t) \). Second, \( F_1(t) \) must satisfy Bayes’ rule whenever possible.

Proposition 2 exhibits a PBE in which the informed suppliers exploit their informational advantage to extend trade credit to their most important customers – the safer firms – while the bank takes advantage of its lower cost of funds to provide financing for the riskier firms. Under a relatively mild assumption, this equilibrium is unique.\(^{11}\)

**Proposition 2** There exists a Perfect Bayesian Equilibrium in which a cut-off type \( \bar{t} \in (t, 1) \) determines whether trade credit or bank loans finance purchases of inputs. Firms with type \( t \in [\bar{t}, 1] \) purchase inputs by borrowing \( I^*(r^B) \) from the bank at the interest rate \( 1 + r^B = \frac{1 + i}{pF_1[\bar{t}]} + \frac{(1+i)F_0(\bar{t})}{p \int_s^\bar{t} f_0(s)ds} \), which is consistent with a belief \( F_1(t) = \text{Prob}(s \leq t | t \leq \bar{t}) \). In turn, any \( t \in [\bar{t}, 1] \) accepts a trade credit offer to finance \( I^*(r^t) \) at the interest rate \( r^t \). Moreover, there exists \( \hat{t} \in [0, 1] \) such that \( r^t = r^B \) if and only if \( t \in [\bar{t}, \hat{t}] \). If \( t > \max\{\bar{t}, \hat{t}\} \), then the optimal interest rate is implicitly defined by \( \epsilon(r^t) = \bar{\epsilon}(r^t, t) \), with \( r^t \) strictly decreasing in \( t \). Provided that \( \frac{d}{dt} \left( \frac{F_0(t)}{f_0(t)} \right) > \frac{c}{1+i} \), the cut-off type \( \bar{t} \) is unique and so is the equilibrium of the game.

In Proposition 2, the equilibrium trade credit contracts follow trivially from the optimality restrictions of Proposition 1: the optimal interest rate is characterized by \( \epsilon(r^t) \leq \bar{\epsilon}(r^t, t) \), and the safest type that has access to trade credit is \( \bar{t} \leq \frac{1+i+c}{p(1+r^B)} \). Likewise, the principal amount of the bank loan, \( I^*(r^B) \), maximizes the borrower’s net output (as in Section 2.4), while Bayes’ rule and the cutoff type \( \bar{t} \in (t, 1) \) pin down the updated belief, \( F_1(t) = F_0(t)/F_0(\bar{t}) \),

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11The sufficient condition for uniqueness is slightly stronger than the monotone hazard condition; the hazard rate must increase with \( t \) at a pace faster than \( c/(1+i) \). Under Assumption 1, the condition \( \frac{d}{dt} \left( \frac{F_0(t)}{f_0(t)} \right) > \frac{c}{1+i} \) holds if \( F_0(t) \) is the C.D.F. of a uniform distribution in the interval \([t, 1]\).
that ultimately determines the interest rate that leaves the bank with zero expected expected profits, that is, 
\[ r^B = \frac{(1+i)F_0(\hat{t})}{p^T\int_0^t s f_0(s)ds} - 1. \]

The zero-profit condition for the bank is a key to understand the cutoff \( \tilde{t} \) that separates the firms that borrow from the bank from those that use trade credit. In equilibrium, the bank’s expected profit with loans to the safer firms just offsets its expected loss with loans to the riskier firms. The interest rate \( r^B \), therefore, falls short of the informed suppliers’ break-even rate for trade credit offers to the riskier types. Accordingly, informed suppliers restrict trade credit offers to firms whose risk of credit is lower than the typical firm that borrows from the bank. In particular, the risk of the cutoff type \( \tilde{t} \) is sufficiently low to make the bank rate offset not only its expected default but also the higher cost of funds of the informed suppliers.

Given \( \tilde{t} \), the cutoff \( \hat{t} \) splits the types with access to trade credit into two groups. The safer ones, \( t > \hat{t} \), pay an interest rate \( r^t \) that decreases with their probability of success. The riskier types, \( t > \max\{\hat{t}, \tilde{t}\} \), pay the interest rate \( r^B \) that keeps them indifferent between trade credit and bank loans. Except for corner solutions, \( \hat{t} \) is implicitly defined by \( \epsilon(r^B) = \tau(r^B, \hat{t}) \), which implies that the informed suppliers’ marginal gains of increasing interest rates above \( r^B \) are equal to their marginal cost of reducing the type-\( \hat{t} \)’s willingness to borrow. If all types pay \( r^B \) then \( \hat{t} = 1 \geq t \), while \( \hat{t} < \tilde{t} \) if \( r^t < r^B \), for any \( t \).

3.3 The likelihood that the cost of trade credit varies with firm characteristics

Typically, interest rates in the trade credit markets vary across industries but not with firm characteristics. As such, a test with power to reject our model should be centered around implications that help predict when or where the cost of trade credit varies with firm characteristics. This section shows that the cost of trade credit is more likely to vary with firm characteristics in financially-distressed industries and in industries with informed suppliers that have a low cost of funds.

To link the cost of trade credit to financial distress, our first task is to extend the model to create classes of risk. As such, split the continuum of firms \( C \) in two sets, \( A_0 \) and \( A_1 \), such that \( A_0 \cup A_1 = C \) and \( A_0 \cap A_1 = \emptyset \). For simplicity, the support of types in the two
sets is still the interval \([t, 1]\), but the cumulative distribution varies across the sets. While the distribution of types in the set \(A_0\) remains \(F_0(t)\), the distribution in \(A_1\) is \(G_0(t)\), with density \(g_0(t)\). We assume that the hazard rate of \(G_0(t)\) is always smaller than \(F_0(t)\), that is,

\[
\frac{g_0(t)}{G_0(t)} < \frac{f_0(t)}{F_0(t)} \quad \text{for any } t \in (t, 1),
\]

which implies that the expected probability of success is smaller in \(A_1\) than in \(A_0\).

From now on, we shall denote the sets \(A_0\) and \(A_1\) as, respectively, the \(F\)-class and the \(G\)-class. The uninformed lenders know which firms are in each class, but cannot distinguish the types of firms in the same class of risk.

Proposition 3 below shows that the greater risk of the \(G\)-class carries through to the equilibrium in the credit markets. In particular, firms in the \(G\)-class pay a higher interest rate to the bank than firms in the \(F\)-class. More interestingly, Proposition 3 shows that, relative to the \(F\)-class, more firms in the \(G\)-class have access to trade credit and their cost of trade credit is more likely to vary with firm characteristics.

**Proposition 3** Assume that the distributions \(F_0\) and \(G_0\) satisfy the inequality (9). Thus, \(\bar{t}^G < \bar{t}^F\) and \(\hat{t}^G < \hat{t}^F\), where \(\bar{t}^k\) is the riskiest type in the class \(k \in \{F, G\}\) that has access to trade credit, and \(\hat{t}^k\) is the safest type in the class \(k\) whose cost of trade credit does not vary with firm characteristics. Moreover, firms in the \(G\)-class pay a higher interest rate to the bank than firms in the \(F\)-class.

The bank’s response to the higher probability of default in the \(G\)-class is quite obvious: It increases its interest rate. In turn, the informed suppliers take advantage of the higher bank rate to selectively raise the cost of trade credit for the riskier firms in the \(G\)-class. By doing so, the informed suppliers preserve the volume of trade credit for the safer firms, while offering trade credit at costlier terms to the riskier firms.\(^{12}\)

Proposition 3 suggests that the cost of trade credit is more likely to vary with firm characteristics in industries that are plagued by financial distress. This implication of our model

\(^{12}\)An increase in risk does not necessarily increase the expected profit of the informed suppliers; additional restrictions on the riskier distribution \(G(t)\) are needed to determine whether the greater leeway to raise the cost of trade credit outweighs the greater probability that trade credit is not paid.
is consistent with Wilner (2000), who argues that suppliers have incentives to bail-out financially distressed customers in order to preserve long-term business relationships. Anticipating their own incentives, suppliers should embed the expected cost of the potential bail-out in the terms of trade credit. Clearly, this expected cost varies with the customer’s risk and is more likely to be relevant in distressed industries.

Of course, suppliers of inputs may also become financially distressed. One might then wonder whether financially-distressed suppliers are more prone to vary the cost of trade credit with firm characteristics. To answer this question, our starting point is that financially-distressed suppliers have difficulties to raise the necessary funds to provide trade credit. In our model, these difficulties map into a higher cost of funds, whose impact on the supply of trade credit is summarized by Proposition 4.

**Proposition 4** *An increase in the suppliers’ cost of funds raises the cutoff types $\bar{t}$ and $\hat{t}$, reducing access to trade credit while lowering (possibly weakly) the fraction of firms whose cost of trade credit falls with the probability of success.*

The intuition for Proposition 4 lies in the trade off between margin of profits and volume of loans. A higher cost of funds decreases the expected margin of trade credit, regardless of the borrower’s firm-specific characteristics. The lower margin of profits makes the volume of trade credit less important for the informed suppliers, weakening their incentive to use the interest rate as an instrument to discriminate the demand for loans. Hence, suppliers with a higher cost of funds provide less trade credit ($\bar{t}$ increases) and make the cost of trade credit less sensitive to firm-specific characteristics ($\hat{t}$ increases).

Arguably, small suppliers of inputs have a higher cost of funds and are more prone to financial distress. If so, Proposition 4 predicts that small suppliers are less inclined to extend trade credit and, when they do so, the terms of trade credit are less likely to vary with the borrower’s credit risk.

To illustrate Proposition 4, assume that $F_0(t)$ is uniformly distributed in the interval $[\bar{t}, 1]$ and consider the production function $Q(I) = I^\alpha$, which implies an iso-elastic demand for

\[13\text{Rajan and Zingales (1995), among others, find evidence that bigger firms have a larger debt capacity.} \]
inputs: \( \epsilon(r) = \frac{1}{1-\alpha} \), for any interest rate \( r \). This production function satisfies our technological assumptions if \( \alpha \in (0, 1) \).

From Proposition 2, the riskiest type that has access to trade credit is implicitly defined by \( \bar{t} = 1 + i + c \frac{1}{1 + i + c} E_1[t] \).\(^{14}\) If \( F_0(t) \) is uniformly distributed, then the expected type of a firm that borrows from the bank is \( E_1[t] = E[t|t \leq \bar{t}] = \frac{t + \bar{t}}{2} \), which implies

\[
\bar{t} = \frac{1 + i + c}{1 + i - c} \bar{t}.
\]  

Equation (10) confirms that a higher cost of funds makes trade credit less pervasive, that is, \( \bar{t} \) increases with \( c \). To see that the cost of trade credit becomes less sensitive to firm characteristics as \( c \) gets larger, assume that \( \alpha > \frac{1 + i + c}{p(1 + r_B)} = t \left( \frac{1 + i + c}{1 + i - c} \right) \) to ensure an equilibrium in which the cost of trade credit may vary with firm characteristics. From Proposition 2, the safest type that pays \( r_B \), i.e. \( \hat{t} \), is implicitly defined by \( \epsilon(r_B) = \tilde{\epsilon}(r_B, \hat{t}) \). In the iso-elastic case, this condition becomes \( \frac{1}{1 - \alpha} = \tilde{\epsilon}(r_B, \hat{t}) \), or equivalently

\[
\hat{t} = \frac{1 + i + c}{p \alpha (1 + r_B)} = \frac{1}{\alpha} \left( \frac{1 + i + c}{1 + i} \right) E_1[t] = \frac{1}{\alpha} \left( \frac{1 + i + c}{1 + i - c} \right) t.
\]  

which proves that \( \hat{t} \) increases with \( c \) because Assumption 1 implies that \( 1 + i > c \).

4 Trade credit and moral hazard problems

Burkart and Ellingsen (2004) argue that trade credit is a pervasive source of external financing because loans in kind are less vulnerable to moral hazard problems. This section demonstrates that although moral hazard may be an important reason for trade credit to exist, it doesn’t explain why the cost of trade credit typically does not vary with firm characteristics.

4.1 The moral hazard problem

To introduce a moral hazard problem in the model, we assume that shareholders may unduly divert part of the cash-flow generation that should be used to pay debt obligations.

\(^{14}\)To obtain \( \bar{t} = \frac{1 + i + c}{1 + i} E_1[t] \), plug \( r_B = \frac{1}{p E_1[t]} - 1 \) into \( \frac{1 + i + c}{p(1 + r_B)} \).
There are two constraints on the shareholders’ opportunistic behavior. First, we do not let them grab corporate assets and run: Diversion of cash is bounded by the firm’s cash-flow generation. Second, debt holders may sue the shareholders of firms that do not pay their debt obligations, unless the court believes that the breach of the debt contract was the unfortunate outcome of a risky but reasonable investment. To curb the risk of prosecution, firms must invest in a diversion technology that makes it more difficult for outsiders to detect opportunistic behavior. If firms pay $B$, then their shareholders can safely capture an amount $\rho B$ of the cash-flow generation. We assume that $\rho \in \left[0, \frac{1+i}{p}\right]$, which implies that the expected return on the diversion technology does not cover the time value of the money, regardless of the firm’s type. Diverting cash is therefore socially inefficient.

To characterize the incentives to divert cash, suppose that a type-$t$ firm borrows $I$, promising to repay $A(I)$ when the project’s cash flow realizes. After receiving the proceeds of the loan, the borrower chooses how much to invest in the project and how much to invest in the diversion technology. Investing $B$ in the diversion technology, on the one hand, lets the shareholders capture an amount $\rho B$ of the firm’s cash-flow generation, whether the debt holders are paid or not. On the other hand, it reduces the cash-flow generation from the project by $Q(I - B) - Q(I - B^*)$. The optimal diversion policy solves

$$\max_{B \in [0, I]} p I \left[ \max \left\{ Q(I - B) - A(I), 0 \right\} + \rho B \right].$$  

(12)

Lemma 1 characterizes the optimal investment in the diversion technology.

**Lemma 1** Let $\bar{B}(I)$ be the largest investment in the diversion technology that does not harm the debt holders, that is, $Q(I - \bar{B}(I)) = A(I)$. Thus, the optimal diversion policy is

$$B(I, \rho) = \begin{cases} 
0 & \text{if } Q'(I) \geq \rho \text{ and } \frac{Q(I)}{I} \geq \rho + \frac{A(I)}{I - \bar{B}^*}, \\
B^* \in (0, \bar{B}(I)) & \text{if } Q'(I - B^*) = \rho \text{ and } \frac{Q(I - B^*)}{I - B^*} \geq \rho + \frac{A(I)}{I - B^*}, \\
I & \text{otherwise}. 
\end{cases}$$  

(13)

The optimal diversion policy, $B(I, \rho)$, takes into account the two conflicting forces underlying the decision to divert cash: The efficiency of the project and the shareholders’ gains from extracting value from the debt holders. These two forces are summarized by the average productivity of the project, $\frac{Q(I)}{I}$, and two marginal productivities; one for the project, $Q'(\cdot)$,
and another for the diversion technology, \( \rho \). The conditions on the marginal productivities ensure that marginal deviations from \( B(I, \rho) \) do not increase the shareholders’ expected payoff. And the lower bounds on the average productivity imply that the project’s expected return is sufficiently large to dissuade the shareholders from diverting all available funds.

The optimal diversion policy suggests that the bank may constrain its supply of loans to eliminate the agency costs of debt. By constraining the loan amount, the bank lowers the scale of the project that the borrowers can undertake, raising its average productivity to a level that dissuades the borrowing firms from investing in the diversion technology. To be sure, constraining the supply of loans imposes costs. Still, Proposition 5 shows that, in equilibrium, the bank designs a loan contract that prevents any diversion of corporate funds.

**Proposition 5** In equilibrium, the bank designs a standard debt contract \((I, A(I))\) that dissuades the borrowers from diverting corporate resources, that is, \( B(I, \rho) = 0 \).

Proposition 5 implies that the optimal debt contract must satisfy three conditions. First, firms must be willing to borrow from the bank (the participation constraint). Second, the debt contract must induce the borrowers to invest all proceeds of the loan in the project (the incentive-compatibility constraint). And third, the debt contract must imply zero expected profits for the bank (the zero-profit condition).

Consider first the zero-profit condition while assuming that the other two constraints hold. As in section 3, a firm’s request of a loan updates the bank’s prior on the borrower’s type to \( F_1(t) \). If the bank correctly believes that the borrower will use the proceeds of the loan to invest in the project, then its expected profit is \( pE_1[t]A(I) - (1 + i)I \), where \( E_1[t] = \int_1^t tdF_1(t) dt \) is the expected type of the borrower. The zero-profit condition implies that the promised return of the loan is \( 1 + r^B = \frac{1 + i}{pE_1[t]} \), exactly as in the equilibrium interest rate that ignores agency costs of debt (see equation (1)). As a result, we can write the loan contract as a pair \((I, r^B)\).

To obtain the loan amount \( I \), we move on to the incentive conditions that assure that the borrower is better off undertaking the project rather than diverting the proceeds of the loan. From Lemma 1, incentive compatibility requires that neither the marginal productivity nor the average productivity of the investment is too small. As Proposition 6 shows, the lower bound
on the average productivity of investment is the relevant restriction for the characterization of the equilibrium loan contract.

**Proposition 6** The equilibrium loan contract is \((I^*(r^B), r^B)\) if and only if \(I^*(r^B) \leq \bar{I}^B(r^B, \rho)\), where \(r^B = \frac{1+i}{1+i}r\), \(I^*(r^B)\) is the optimal investment in the project given \(r^B\), and \(\bar{I}^B(r^B, \rho)\) is implicitly defined by

\[
Q(\bar{I}^B(r^B, \rho)) = \rho + (1 + r^B).
\]

(14)

If \(I^*(r^B) > \bar{I}^B(r^B, \rho)\), then the equilibrium loan contract is \((\bar{I}^B(r^B, \rho), r^B)\), with \(\frac{\partial \bar{I}^B(r^B, \rho)}{\partial \rho} < 0\) and \(\frac{\partial \bar{I}^B(r^B, \rho)}{\partial r^B} < 0\).

The intuition for Proposition 6 lies on the marginal productivity of the diversion technology, \(\rho\). If \(\rho\) is close to zero, then the gains from diverting cash are small, making it unprofitable for the shareholders to sacrifice resources that could be used in the project. The moral hazard problem is therefore irrelevant: There is no credit constraint, and the bank offers the debt contract \((I^*(r^B), r^B)\) that finances the optimal scale of the project.

As \(\rho\) increases, the gains from diverting cash grow, raising the minimum productivity of the project that dissuades the shareholders from diverting corporate resources (see Lemma 1). Since the production function is concave, a necessary condition for the average productivity to increase is that the investment in the project falls. Hence, as \(\rho\) gets bigger, the largest investment in the project that satisfies the productivity condition, \(\bar{I}^B(r^B, \rho)\), decreases.

Credit constraint arises when the benefit of diverting cash, \(\rho\), gets sufficiently large to make the bank offer a loan amount, \(\bar{I}^B(r^B, \rho)\), that does not finance the optimal scale of the project. The cutoff for \(\rho\) that implies credit constraint is defined by

\[
\bar{I}^B(r^B, \rho^\text{const}(r^B)) = I^*(r^B).
\]

If \(\rho \leq \rho^\text{const}(r^B)\) then \(\bar{I}^B(r^B, \rho) \geq I^*(r^B)\), implying that the bank can offer the first best contract, \((I^*(r^B), r^B)\), without fearing agency costs of debt. Moral hazard is costly, though, if \(\rho > \rho^\text{const}(r^B)\). In this case, the bank offers the constrained contract \((\bar{I}^B(r^B, \rho), r^B)\) that keeps the borrower as close as possible to the optimal scale of the project, without eliciting incentive for diversion of cash. We can thus write the value of a type-\(t\) firm that borrows from
the bank as

\[ p^t \Pi^B(r^B, \rho) \equiv \begin{cases} 
   p^t \left[ Q(I^*(r^B)) - (1 + r^B)I^*(r^B) \right] & \text{if } \rho \leq \rho^{const}(r^B), \\
   p^t \left[ Q(\bar{I}^B(r^B, \rho)) - (1 + \bar{r})\bar{I}^B(r^B, \rho) \right] & \text{if } \rho > \rho^{const}(r^B). 
\end{cases} \quad (15) \]

Note that equation (15) does not provide a complete characterization of the value of a firm that borrows from the bank, because \( r^B \) depends on the borrower’s expected type \( (E_1[t]) \). The next section obtains \( E_1[t] \) as part of the equilibrium of the game.

### 4.2 Equilibrium with moral hazard problems

As Burkart and Ellingsen (2004) point out, loans in kind – like trade credit – do not give leeway for insiders to divert corporate assets. We assume, therefore, that suppliers do not fear opportunist behavior. And yet, this section demonstrates that severe moral hazard problems in the market for bank loans make trade credit more pervasive and increase the likelihood that the cost of trade credit varies with firm characteristics.

A first step to determine the impact on the trade credit markets of the bank’s moral hazard problems is to characterize the informed suppliers’ opportunity set. Without moral hazard, the highest interest rate that informed suppliers can charge is the bank rate \( r^B \).\(^{15}\) Moral hazard may relax this upper bound, because credit-constrained firms are willing to pay a premium for larger credit lines. The largest interest rate that informed suppliers can charge – call it \( \bar{r} \) – keeps the credit-constrained firms indifferent between a cheaper (but undersized) bank loan and a larger (but costlier) trade credit contract, that is,\(^{16}\)

\[ p^t \left[ Q(I^*(\bar{r})) - (1 + \bar{r})I^*(\bar{r}) \right] = p^t \Pi^B(r^B, \rho). \quad (16) \]

The left-hand side of equation (16) is the expected profit of a type-\( t \) firm that invests in the project, after accepting trade credit at the interest rate \( \bar{r} \). The right-hand side is the

\(^{15}\)In this section, we continue assuming that the problem of the informed suppliers is to design linear trade credit contracts that maximize expected financial profits.

\(^{16}\)Severe moral hazard problems may break down the market for bank loans. In this case, moral hazard cannot explain the cross-sectional variation of the cost of trade credit because, in our model, suppliers are insulated from moral hazard problems, whether they have an informational advantage or not. Accordingly, we assume that a bank loan is always the firms’ outside option to trade credit offered by their main suppliers.
expected profit of a type-\( t \) firm that borrows from the bank. Using equation (15), one can check that \( \bar{r} = r^B \), if moral hazard problems do not imply credit constraint in the market for bank loans. Otherwise, \( \bar{r} > r^B \). We have thus established:

**Lemma 2** The cost of trade credit cannot be higher than the interest rate \( \bar{r} \) that is implicitly defined by equation (16). If the firm is not credit constrained, i.e. \( \rho \leq \rho^{\text{const}}(r^B) \), then \( \bar{r} \) is the bank rate \( r^B \). Otherwise, \( \bar{r} > r^B \).

From Lemma 2, we can write the problem of the informed suppliers as

\[
\max_{r^t} \left( p^t (1 + r^t) - (1 + i + c) \right) I^*(r^t) \tag{17}
\]

subject to \( \frac{1 + i + c}{p^t} - 1 \leq r^t \leq \bar{r} \).

The objective function (17) is the expected profit of an informed supplier that extends trade credit to a type-\( t \) firm at the interest rate \( r^t \). The interest rate \( r^t \) must be larger than the break-even point, \( \frac{1 + i + c}{p^t} - 1 \), but it is bounded by the largest interest rate \( \bar{r} \) that the firm is willing to pay. The upper bound on the interest rate implies that it is profitable for informed suppliers to offer trade credit if and only if the borrower’s type is

\[
t \geq \frac{1 + i + c}{p(1 + \bar{r})}. \tag{18}
\]

A straightforward comparison of Programs (5) and (17) shows that they are equal, unless the moral hazard problems – parameterized by \( \rho \) – let the informed suppliers raise the cost of trade credit above the bank rate. Accordingly, Proposition 7 identifies a cutoff value for \( \rho \) below which the moral hazard problems have no impact on the trade credit markets.

**Proposition 7** There exists \( \rho^{\text{const}} > 0 \) such that \( \rho \leq \rho^{\text{const}} \) implies that the strategies and belief of Proposition 2 form a Perfect Bayesian Equilibrium. In this equilibrium, which is unique if \( \frac{d}{dt} \left( \frac{F_0(t)}{f_0(t)} \right) > \frac{c}{1 + i} \), moral hazard problems are irrelevant; the bank induces the borrowing firms to invest in the project, without imposing credit constraint. In contrast, \( \rho > \rho^{\text{const}} \) implies that there is no equilibrium without credit constraint in the market for bank loans.
Proposition 7 shows that there is no equilibrium without credit constraint, if the shareholders’ gains from diverting cash are larger than $\rho_{\text{const}}$. As it turns out, it is easy to exhibit technologies that imply credit constraint in the market for bank loans. It can be shown, for example, that the iso-elastic production function, $Q(I) = I^\alpha$, implies that $\rho_{\text{const}} \in \left(0, \frac{1+i}{p}\right)$, if $\alpha \in \left(\frac{1}{1+t}, 1\right)$. For any $\rho \in \left(\rho_{\text{const}}, \frac{1+i}{p}\right)$ and $r^B = \frac{1+i}{pE[\ell]}$, the optimal unconstrained investment, $I^*(r^B)$, is strictly bigger than the largest loan $\bar{I}^B(r^B, \rho)$ that induces the borrowers to invest in the project. Proposition 6 thus implies that the equilibrium debt contract, $(\bar{I}^B(r^B, \rho), r^B)$, does not let the borrowers implement the optimal scale of the project.

Credit constraint does not change the essence of the suppliers’ problem, though. In particular, the optimal interest rate $r^t$ still solves a trade off between margin of profit and volume of trade credit. Analogously to Proposition 1, the first order condition of Program (17) implies that $r^t$ is optimal for the informed supplier if and only if $\epsilon(r^t) \leq \bar{\epsilon}(r^t, t) \equiv \frac{tp(1+r^t)}{tp(1+r^t) - (1+i+c)}$. And a cutoff $\hat{t}$ determines which types pay the interest rate $\bar{r}$, defined in equation (16), that keeps the borrowers indifferent between trade credit and a bank loan. If $t \leq \hat{t}$, then $r^t = \bar{r}$. If $t > \hat{t}$, then the optimal interest rate is implicitly defined by $\epsilon(r^t) = \bar{\epsilon}(r^t, t)$, with $\frac{dr^t}{dt} < 0$. Hence, a necessary and sufficient condition for the cost of trade credit not to vary with firm characteristics is $\hat{t} = 1$, or equivalently, $\epsilon(\bar{r}) \leq \bar{\epsilon}(\bar{r}, 1)$.

To integrate the optimal trade credit contracts with the market for bank loans, we proceed as in Section 3.2 and consider the following strategies and belief

$$E(\rho) = \left\{ \left(\bar{I}^B(r^B(\rho), \rho), r^B(\rho)\right), \left(I^*(r^t(\rho)), r^t(\rho)\right) \mid t \in [\bar{t}(\rho), 1], \left\{1_{\text{bank}}\right\} \mid t \in [\ell, 1], F_1(t|\bar{I}(\rho)) \right\}. \quad (19)$$

$E(\rho)$ is a Perfect Bayesian Equilibrium (PBE) if there exists $\hat{t}(\rho) \in [\bar{t}, 1)$ such that: (i) The debt contract $\left(\bar{I}^B(r^B(\rho), \rho), r^B(\rho)\right)$ ensures to the bank zero expected profits under the belief $F_1(t|\bar{I}(\rho))$, while keeping the scale of the project as close as possible to the first best without inducing the borrowers to divert cash; (ii) it is optimal for the informed suppliers to lend $I^*(r^t(\rho))$ to any type $t \geq \hat{t}(\rho)$, where $r^t(\rho)$ is the interest rate that solves Program (17); (iii) firms choose the financing alternative that maximizes their expected profits; and (iv) Bayes’ rule determines $F_1(t|\bar{I}(\rho))$ from the equilibrium strategies, whenever possible. Proposition 8 shows that, for any $\rho \in (\rho_{\text{const}}, \frac{1+i}{p})$, there exist $\bar{t}(\rho) \in [\ell, 1)$ that makes $E(\rho)$ a PBE.
Proposition 8 For any \( \rho \in (\rho_{\text{const}}, \frac{1+i}{p}) \), there exists \( \bar{t}(\rho) \in [t, 1] \) that makes \( E(\rho) \) a PBE in which the cost of trade credit is \( r^t(\rho) = \bar{r}(\rho) \) for any \( t \in [\bar{t}(\rho), \hat{t}(\rho)] \) and some \( \hat{t}(\rho) \in [0, 1] \). If \( t > \max\{\bar{t}(\rho), \hat{t}(\rho)\} \), \( r^t(\rho) \) is defined by \( \epsilon(r^t(\rho), t) = \bar{\epsilon}(r^t(\rho), t) \), with \( r^t(\rho) < \bar{r}(\rho) \) and \( \frac{d\bar{r}(\rho)}{dt} < 0 \). Regardless of \( t \), \( \frac{d\bar{r}(\rho)}{dp} > 0 \). Moreover, there exists \( \rho^{\text{all}} > \rho_{\text{const}} \) such that, for any \( \rho \in (\rho_{\text{const}}, \rho^{\text{all}}) \), \( \bar{t}(\rho) \in (t, 1) \), \( \frac{d\bar{r}(\rho)}{dp} < 0 \), \( F_1(t|\bar{t}(\rho)) = \text{Prob}(s \leq t|t \leq \bar{t}(\rho)) \), and \( r^B(\rho) = \frac{F_0(\bar{t}(\rho))(1+i)}{p \int_{1}^{\bar{t}(\rho)} s f_0(s) ds} \). This equilibrium is unique, provided that \( \frac{d}{dt} \left( \frac{F_0(t)}{F_0(\bar{t})} \right) > \frac{c}{1+i} \). If \( \rho \in [\rho^{\text{all}}, \frac{1+i}{p}] \), then \( E(\rho) \) is the unique equilibrium that satisfies the consistency requirement of Kreps and Wilson (1982). In this equilibrium, \( \bar{t}(\rho) = \frac{1}{2}, F_1(t|\bar{t}(\rho)) = 1 \) for any \( t \in [t, 1] \) and \( r^B(\rho) = \frac{1+i}{pt} \).

Consistently with Burkart and Ellingsen (2004), the equilibrium of Proposition 8 predicts that moral hazard makes trade credit more pervasive (\( \frac{d\bar{r}(\rho)}{dp} < 0 \)), until \( \rho \) crosses a cutoff \( \rho^{\text{all}} \) that rules out bank loans, that is, \( \bar{t}(\rho) = \frac{1}{2} \), for any \( \rho \geq \rho^{\text{all}} \). The reason for moral hazard to expand trade credit is quite obvious. Agency costs of debt raise the bank rate, increasing the firms’ willingness to pay for trade credit (i.e., \( \frac{d\bar{r}(\rho)}{dp} > 0 \)). As expected, a greater willingness to pay for trade credit strengthens the suppliers’ incentives to offer trade credit.

The link between moral hazard problems and the cost of trade credit sheds some light on a puzzling finding of Ng, Smith and Smith (1999), whose survey of trade credit policies suggests that the cost of trade credit rarely falls when banks lower interest rates. In our model, the riskless rate, \( \bar{r} \), is the source of economy-wide shocks in the bank rate. Although the direct effect of a lower riskless rate is to reduce the cost of trade credit, Mateut, Bougheas and Mizen (2006) argue that counter-cyclical monetary policies imply a positive correlation between low interest rates and economic distress. As Jensen and Meckling (1976) point out, economic distress increases the agency costs of debt.\(^{17}\) From Proposition 8, larger agency costs of debt (i.e., higher values of \( \rho \)) raise the cost of trade credit (\( \frac{d\bar{r}(\rho)}{dp} > 0 \)), possibly canceling the downturn pressure due to a lower riskless rate.

Perhaps more interestingly, Proposition 9 below shows that moral hazard problems increase the sensitivity of the cost of trade credit with respect to the borrower’s type. Moral hazard,\(^{17}\)In principle, financially distressed borrowers could create moral hazard problems not only for the banks but also for their suppliers. Nonetheless, Cuñat (2002) argues that suppliers are less vulnerable to moral hazard problems, because they can threaten to stop supplying vital intermediate goods. In this case, the suppliers are likely to be spared of moral hazard problems, making the banks the main targets of opportunistic behavior.
therefore, does not explain why the cost of trade credit rarely varies with firm characteristics.

**Proposition 9** Let $\hat{t}(\rho) \in [\bar{t}(\rho), 1]$ be the safest type that pays the interest rate $\bar{r}(\rho)$. Thus, $\hat{t}(\rho)$ decreases with $\rho$ and there is a cutoff value, $\bar{\rho}$, such that $\rho \leq \bar{\rho}$ implies that, in equilibrium, the cost of trade credit never varies with firm characteristics (i.e., $\hat{t}(\rho) = 1$). If $\rho > \bar{\rho}$, then the cost of trade credit decreases with the probability of success for any $t \geq \hat{t}(\rho)$.

Intuitively, moral hazard weakens the bank’s ability to finance the project, allowing the informed suppliers to fetch higher margins in the trade credit transactions. As we have already argued, larger margins make the volume of trade credit more important for the informed suppliers, strengthening their incentive to vary the interest rates with the borrowers’ risk of credit. Accordingly, Proposition 9 shows that a larger $\rho$ increases the set of types whose cost of trade credit falls with the probability that the firm succeeds.

## 5 Discussion

### 5.1 Relationship lending

Suppliers are not the only informed lenders. Banks can build lending relationships that give them private information about their preferred customers’ risk. Competition between the informed supplier and the informed bank may drive down the interest rates to the informed supplier’s break even point, $\frac{1+i+c}{\rho'} - 1$, which depends on firm specific risk factors.

Nonetheless, there are costs for banks to engage in relationship lending. For a bank to invest in a lending relationship, it must start offering premium services (or low interest rates), in exchange for the customer’s promise (possibly implicit) of future profitable deals. Arguably, banks are more confident about these promises in concentrated credit markets, which give fewer opportunities for borrowers to shop around for the least costly provider of their financial needs. Consistently with this conjecture, Petersen and Rajan (1995) show that, in the U.S., relationship lending is more pervasive in concentrated credit markets.

In the context of our model, the findings of Petersen and Rajan (1995) imply that there are stronger incentives for the bank to engage in relationship lending with types that do not
have access to trade credit, that is, any \( t \in [\hat{t}, \tilde{t}) \). With these firms, the bank does not have to lower interest rates to preempt trade credit from an informed supplier. The likelihood that relationship lending gives rise to a second informed lender is therefore higher when it has no impact on the cost of trade credit, that is, when the bank engages in a relationship with a firm that the supplier has no interest in extending trade credit. If so, allowing for banks to invest in relationship lending does not unravel Proposition 2. And the cost of trade credit does not vary with firm characteristics whenever the demand for inputs is sufficiently inelastic.

5.2 Trade credit with alternative outside options

In the basic model, trade credit does not vary with firm characteristics because the supplier competes with uninformed lenders. It is plausible, nonetheless, that the relevant outside option of a firm is a sale of assets, rather than borrowing from an uninformed lender.

For example, consider the impact on the airline industry of an increase in the price of fuel. In response to the price increase, an airline may want to buy planes that consume less fuel or, alternatively, reduce its fleet. While the return on the older planes certainly depends on the company’s idiosyncratic ability to schedule flights, purchase inputs, and negotiate salaries, these firm-specific characteristics are not so important for the resale values of the airplanes, which are mainly determined by the economic conditions of the airline industry. Informed suppliers may thus want to raise the cost of trade credit until the airlines are indifferent between the two reorganization plans: buying new planes or reducing the fleet. If so, the cost of trade credit will not vary significantly with the firms’ characteristics.\(^{18}\)

5.3 Incentive for suppliers to disclose private information

Banks often seek information from suppliers about the payment history of their customers. If suppliers answer these inquiries truthfully, then their informational advantage disappears and so does the reason we give for the cost of trade credit not to vary with firm characteristics, that is, competition with uninformed lenders.

\(^{18}\)Felli and Harris (1996) explore the role of outside options in a model of investment in human capital.
To investigate the suppliers’ incentives to disclose information, consider the firms that pay the interest rate that keeps them indifferent between trade credit and loans from uninformed lenders. In our model, the supplier does not raise the cost of trade credit for these firms because the uninformed lenders provide them with an outside option. If the informed supplier lets a bank know that the market mistakenly perceives these firms as overly risky, then the bank rates go down, forcing the informed supplier to reduce the cost of trade credit. Hence, suppliers may not have the proper incentive to disclose private information that enhances firms’ credit standings.

Suppliers have incentive to disclose information, though, if they are credit constrained. In this case, proving that their customers have solid credit standings helps the suppliers obtain the necessary funds to offer trade credit. Note, however, that an incentive-compatible revelation mechanism is needed for the credit-constrained suppliers to truthfully report their customers’ credit standing. Without a revelation mechanism, credit-constrained suppliers have incentives to portray to the banks a rosy view about their customers’ financial health.

We are not aware of a study that documents profit-sharing rules or other revelation mechanisms between banks and suppliers in standard trade-credit transactions. It is conceivable, though, that some sort of revelation mechanism is in place in project loans, which are typically structured around complex contracts. If so, the bank internalizes not only the supplier’s private information but also its incentive to extract financial rents from the firm. In particular, there is no reason for the bank to force the supplier to move away from the pricing strategy that keeps the riskier firms indifferent between trade credit and a loan from an uninformed lender. As such, revelation mechanisms should not break down the equilibrium in which trade credit varies across industries, but not with firm characteristics.

5.4 Nonlinear trade credit contracts

Linear trade credit contracts should probably be understood as mechanisms to facilitate repeated interactions among sellers and buyers. To avoid a costly bargaining for credit, suppliers make one-for-all offers that may not extract all of the firms’ rents. Nonetheless, large transactions – like a sale of a commercial jet – should raise incentives for suppliers to design
tailor-made contracts that enhance their ability to extract rents.

We argue below that a richer space of trade credit contracts does not unravel the main insights of our paper, if default imposes small economic costs on the suppliers. To link the cost of trade credit to the cost of default, this section assumes that the investment in the inputs may not be entirely destroyed in case the project fails; failure leaves a residual value $\delta I$, with $\delta \in [0, 1 + i]$. As $\delta$ goes up, the cost of default drops, reaching zero when the salvage value pays for the supplier’s production cost and the time value of money, that is, $(1 + i)I$.19

Efficient debt contracts should finance value-maximizing investment plans. Our first task, therefore, is to solve the maximization program that yields the efficient level of investment of a type-$t$ firm that accepts trade credit:

$$
\max_I p^t \left( Q(I) - (1 + i)I \right) + (1 - p^t) \left( \delta I - (1 + i)I \right).
$$

The first term in the objective function, (20), is the economic reason for a type-$t$ firm to seek financing for the project. With probability $p^t$, the project succeeds and the value added of the investment is $Q(I) - (1 + i)I$. But, with probability $1 - p^t$, the firm runs into problems that destroy the output $Q(I)$, leaving a salvage value $\delta I$. The necessary and sufficient first order condition of problem (20) is

$$
Q'(I^{\text{eff}}) = 1 + i + (1 + i - \delta) \left( \frac{1}{p^t} - 1 \right).
$$

The first-order condition shows the impact of the firm-specific risk factors on the efficient level of investment. In the absence of risk, the efficient investment makes the marginal productivity of investment, $Q'(I^{\text{eff}})$, equal to the marginal social cost, $1 + i$. Firm specific factors matter because riskier firms are more likely to get into problems that imply an economic loss, thereby depressing the efficient investment level. Accordingly, we write the efficient investment, $I^{\text{eff}}(p^t, \delta)$, as a function of the probability that the project succeeds, $p^t$, and the salvage value of the investment in the event that the project fails, $\delta$.

Consider a trade credit contract that lets the type-$t$ firm finance the efficient level of inputs, $I^{\text{eff}}(p^t, \delta)$, in exchange for a payment $A_t$ in the state of nature that the project succeeds. In the default state, the contract stipulates that the supplier captures the residual value of the

---

19The results in Sections 3 and 4 hold for any $\delta \in [0, 1 + i]$. 

27
project, $\delta I^{eff}(p^t, \delta)$. This debt contract is optimal for the informed suppliers, provided that satisfies the following conditions:

$$p^t A_t + (1 - p^t) \delta I^{eff}(p^t, \delta) \geq (1 + i + c) I^{eff}(p^t, \delta), \quad \text{and}$$
$$p^t \left( Q(I^{eff}(p^t, \delta)) - A_t \right) = p^t \Pi_B(r^B, \rho). \quad (23)$$

The inequality (22) ensures that it is profitable for the informed suppliers to extend trade credit to the type-$t$ firm. In addition to being profitable for the suppliers, the optimal contract must keep the borrowers on their reservation values. For this to happen, the borrower’s expected profit under the contract (the left-hand side of equation (23)) must be equal to its expected profit from borrowing from the bank, $p^t \Pi_B(r^B, \rho)$ (see equation (15)).

To determine whether the optimal trade credit contract varies with firm-specific risk factors, assume that inequality (22) holds and solve for $A_t$ in equation (23) to obtain

$$A_t = Q(I^{eff}(p^t, \delta)) - \Pi_B(r^B, \rho). \quad (24)$$

Equation (24) shows that the efficient investment, $I^{eff}(p^t, \delta)$, ties the cost of trade credit, $A_t$, to the firm’s type: economic efficiency dictates that firms with a higher probability of success (i.e., larger $p^t$) invest more, enhancing the supplier’s ability to extract rents. As a result, safer firms pay a higher cost of trade credit.

Nonetheless, the impact of firm-specific risk factors on the optimal trade credit contract may be small. One can check in equation (21) that the probability $p^t$ vanishes from the first order condition of the efficient investment, if default does not impose an economic cost on the supplier, that is, $\delta = 1 + i$. In this case, the cost of trade credit depends only on the industry’s technology, $Q(\cdot)$, and the interest rates set by the uninformed lenders – summarized by $\Pi_B(r^B, \rho)$ – which, by construction, cannot vary with information that is privy to suppliers.

Of course, it is unlikely that suppliers can repossess sold inputs at no cost. However, some simple algebra shows that the sensitivity of the efficient investment with respect to $p^t$ decreases monotonically with the salvage factor, converging to zero as $\delta$ approaches $1 + i$. The model thus predicts that it is harder for empiricists to detect a link between firm characteristics and the cost of trade credit, if default imposes small economic losses on suppliers.
Petersen and Rajan (1997) argue that it is easier for suppliers to transform repossessed inputs (rather than finished goods) into liquid assets. As such, industries with a high fraction of intermediate goods in inventory should be associated with a large salvage value. Assuming that nonlinear trade credit contracts are pervasive in concentrated markets for inputs, our model predicts that, in these markets, a high fraction of intermediate goods decreases the likelihood that the cost of trade credit varies with firm characteristics.

6 Conclusion

In the U.S., several studies have documented that the cost of trade credit varies across industries but not with firm characteristics. At first glance, this finding is hard to reconcile with the evidence that suppliers are informed lenders. After all, basic economic principles suggest that riskier firms should pay higher interest rates.

We argue that competition with uninformed lenders explains why interest rates in the trade credit markets do not seem to vary with firm characteristics. The uninformed lenders do not let the informed suppliers raise the cost of trade credit above the cost of their loans. Because the cost of these loans is likely to vary across industries but not with firm characteristics, the same pattern applies to the cost of trade credit.

Of course, there are other explanations for the perceived lack of cross-sectional variation of the cost of trade credit. Ng, Smith and Smith (1999) argue, for instance, that suppliers may account for firm characteristics by granting discounts for payments after the due date or by selectively reducing the price of the inputs. Alternatively, suppliers may take into account conformity with local practice when setting the cost of trade credit. To be sure, these theories are plausible. However, they do not explain why the terms of trade credit vary across industries. In contrast, our model predicts that interest rates in trade credit markets vary more strongly with firm characteristics in industries that are plagued by economic distress and moral hazard problems. These implications could be the subject of an empirical test.

20 Young and Burke (2001) argue that customs explain the high degree of uniformity of cropsharing contracts.
References


Appendix: Proofs

Proof of Proposition 1: If \( t \geq \tilde{t} = \frac{1+i+c}{r(1+r)} \), then there exists solution to the supplier’s problem because the objective function (5) is continuous and the opportunity set is compact. Assuming \( t \geq \tilde{t} \), differentiate the objective function with respect to \( r^t \) to obtain

\[
\Psi(r^t, p^t) = \frac{\Gamma(r^t)}{1+r^t} \left[ -\epsilon(r^t) \left( p^t(1+r^t) - (1+i+c) \right) + p^t(1+r^t) \right],
\]

where \( \epsilon(r) = -\frac{(1+r)\partial p(r)}{r^t} \). For any \( t \geq \tilde{t} \), simple algebra shows that

\[
\Psi(r^t, p^t) \geq 0 \iff \epsilon(r^t) \leq \frac{tp(1+r^t)}{tp(1+r^t) - (1+i+c)} \equiv \tau(r^t, t),
\]

with \( \Psi(r^t, p^t) = 0 \) if and only if \( \epsilon(r^t) = \tau(r^t, t) \).

Case 1: \( \epsilon(r^B) \leq \tau(r^B, t) \). From (26), \( \Psi(r^B, p^t) \geq 0 \) is equivalent to \( \epsilon(r^B) \leq \tau(r^B, t) \), which holds by assumption. For \( r^t \in \left[ \frac{1+i+c}{p^t} - 1, r^B \right) \), \( \epsilon(r^B) \leq \tau(r^B, t) \) implies \( \epsilon(r^t) < \tau(r^t, t) \) because \( \epsilon(r^t) \) is non-decreasing in \( r^t \) (see Assumption 2) and \( \frac{\partial \tau(r^t, t)}{\partial r^t} < 0 \). It then follows from (26) that \( \Psi(r^t, p^t) > 0 \) for any \( r^t \in \left[ \frac{1+i+c}{p^t} - 1, r^B \right) \). Hence, \( \Psi(r^t, p^t) \geq 0 \), with equality only at \( r^t = r^B \), proving that \( r^B \) is the unique solution to the supplier’s problem.

Case 2: \( \epsilon(r^B) > \tau(r^B, t) \). From (26), \( \epsilon(r^B) > \tau(r^B, t) \) implies \( \Psi(r^B, p^t) < 0 \), proving that \( r^B \) does not satisfy the first order condition. Moreover, \( \frac{1+i+c}{p^t} - 1 \) cannot be optimal either, because it implies zero expected profits while any slightly larger interest rate yields strictly positive expected profits. Hence, \( r^t \in \left( \frac{1+i+c}{p^t} - 1, r^B \right) \) with \( \Psi(r^t, p^t) = 0 \). To see that the optimal interest rate is unique, suppose that there exist \( \hat{r}^t \) and \( r^t \) with \( \hat{r}^t < r^t \) and \( \Psi(r^t, p^t) = \Psi(\hat{r}^t, p^t) = 0 \), which, from (26), implies \( \epsilon(r^t) = \tau(r^t, t) \) and \( \epsilon(\hat{r}^t) = \tau(\hat{r}^t, t) \). A contradiction follows because \( \epsilon(r^t) \) non-decreasing in \( r^t \), \( \tau(r^t, t) \) decreasing in \( r^t \), \( \epsilon(r^t) = \tau(r^t, t) \), and \( r^t < \hat{r}^t \) jointly imply that \( \epsilon(r^t) > \tau(\hat{r}^t, t) \). For the comparative statics of \( r^t \) with respect to \( p^t \), apply the Implicit Function Theorem to \( \Psi(r^t, p^t) = 0 \) to obtain

\[
\frac{dp^t}{dr^t} = -\frac{\partial \Psi(r^t, p^t)}{\partial r^t} = \frac{-\partial \Psi(r^t, p^t)}{\partial p^t} \frac{\Gamma(r^t)}{1+r^t} \left[ -\epsilon(r^t)(1+r^t) + (1+r^t) \right] \leq 0,
\]

with the inequality following from the second order necessary condition for \( r^t \), \( \frac{\partial \Psi(r^t, p^t)}{\partial r^t} \leq 0 \), and \( \frac{\partial \Psi(r^t, p^t)}{\partial p^t} = \frac{\Gamma(r^t)}{1+r^t} \left[ -\epsilon(r^t)(1+r^t) + (1+r^t) \right] < 0 \) because \( 1 < \frac{tp(1+r^t)}{tp(1+r^t) - (1+i+c)} = \tau(r^t, t) = \epsilon(r^t) \).

Hence, \( p^t(r^t) \) is non-increasing, and so is the inverse function \( r^t(p^t) \).
To show that \( r^t(p^t) \) is strictly decreasing in \( p^t \), suppose, by absurd, that there exist \( k, p^f \) and \( p^{f'} \) with \( r^t(p^f) = r^t(p^{f'}) = k \) and \( p^f < p^{f'} \). Since the previous paragraph demonstrated that \( r^t(p^t) \) is non-increasing, it follows from \( r^t(p^f) = r^t(p^{f'}) \) that \( r^t(p^t) = k \) for any \( p^t \in (p^f, p^{f'}) \). Taking the total derivative of \( \Psi(k, p^t) = 0 \) with respect to \( p^t \) yields \( \frac{\partial \Psi(k, p^t)}{\partial p^t} = 0 \), for all \( p^t \in (p^f, p^{f'}) \), contradicting \( \frac{\partial r^t(p^t)}{\partial p^t} < 0 \), for any \( t \in [\bar{t}, 1] \). We thus conclude that \( \frac{dr^t}{dp^t} < 0 \).

**Proof of Proposition 2:** The proof is divided in two parts. The first part proves existence of equilibrium. The second part shows that the equilibrium is unique.

**Part-I (Existence of Equilibrium):** We shall demonstrate that there exist \( \bar{t} \in (t, 1) \) and \( \hat{t} \in [0, 1] \) that make the following strategies and belief a Perfect Bayesian Equilibrium: (i) The bank offers to all firms the debt contract \( (I^*(r^B), r^B = \frac{1+i}{p_{t}^f} F_0(\bar{t}) - 1) \); (ii) the informed suppliers offer \((I^*(r^t), r^t)\) to any \( t \in [\bar{t}, \hat{t}] \). If \( t \in [\bar{t}, \hat{t}] \), then \( r^t = r^B \). If \( t > max\{\bar{t}, \hat{t}\} \), then \( r^t < r^B \), with \( \epsilon(r^t) = \bar{t} \) and \( \frac{dr^t}{dt} < 0 \); (iii) uninformed suppliers do not offer trade credit; (iv) firms borrow from the bank to finance purchases of inputs if trade credit is not available or \( r^t > r^B \); (v) \( F_1(t) = F_0(t)/F_0(\bar{t}) \).

**Belief and the uninformed lenders’ strategy:** Given the proposed strategies, Bayes’ rule implies that the uninformed lenders’ belief conditioned on a loan request is \( F_1(t) = \text{Prob}(s \leq t|t < \bar{t}) = F_0(t)/F_0(\bar{t}) \). Under this belief, section 2.4 shows that it is optimal for the bank to offer credit to all firms at the interest rate \( 1 + r^B = \frac{1+i}{p_{E_1}[\bar{t}]} = \frac{1+i}{p \int^\bar{t} s f_0(s)ds} \). At this interest rate, trade credit is not profitable for uninformed suppliers with a cost of funds \( i + c > i \).

**The strategy of the informed suppliers:** From Proposition 1, it is optimal for informed suppliers to offer trade credit if and only if the borrower’s type is \( t \geq \bar{t} = \frac{1+i+c}{p(1+r^B)} \). Plugging \( 1 + r^B = \frac{(1+i)F_0(\bar{t})}{p \int^\bar{t} s f_0(s)ds} \) into \( \frac{1+i+c}{p(1+r^B)} \) yields \( \bar{t} = \frac{1+i+c}{1+i} \left( \int^\bar{t} s f_0(s)ds \right)/F_0(\bar{t}) \). To prove that there is \( \bar{t} \in (\bar{t}, 1) \) such that \( \bar{t} = \frac{1+i+c}{1+i} \left( \int^\bar{t} s f_0(s)ds \right)/F_0(\bar{t}) \), define \( T(t) = t - \frac{1+i+c}{F_0(\bar{t})} \int^\bar{t} s f_0(s)ds \). Hence, existence of \( \bar{t} \in (\bar{t}, 1) \) is equivalent to existence of a root for \( T(t) \) in the interval \((\bar{t}, 1)\). Note that \( T(t) \) is a continuous function with \( \lim_{t \to 1} T(t) = \bar{t} - \frac{1+i+c}{1+i} t < 0 \) and, by Assumption 1, \( T(1) = 1 - \frac{1+i+c}{1+i} E_0[\bar{t}] > 0 \). It then follows from the Intermediary Value Theorem that there is \( \bar{t} \in (\bar{t}, 1) \) such that \( T(\bar{t}) = 0 \).
Consider any \( t \geq \bar{t} \) and assume that \( \epsilon(r^B) \leq \bar{\epsilon}(r^B, 1) \), which implies that \( \epsilon(r^B) \leq \bar{\epsilon}(r^B, t) \) for any \( t \leq 1 \), because \( \bar{\epsilon}(r^B, t) \) decreases with \( t \). It then follows that \( r^t = r^B \) for any \( t \in [\bar{t}, 1] \) because the proof of Proposition 1 shows that a necessary and sufficient condition for \( r^t \) to be optimal is \( \epsilon(r^t) \leq \bar{\epsilon}(r^t, t) \). To obtain \( r^t = r^B \) for any \( t \in [\bar{t}, \hat{t}] \), just set \( \hat{t} = 1 \). Assume now that \( \bar{\epsilon}(r^B, 1) < \epsilon(r^B) < \bar{\epsilon}(r^B, \bar{t}) \). In this case, \( \frac{\partial \bar{\epsilon}(r^B, \bar{t})}{\partial t} < 0 \) implies that there is \( \bar{t} \) such that \( \epsilon(r^B) = \bar{\epsilon}(r^B, \bar{t}) \), with \( \epsilon(r^B) > \bar{\epsilon}(r^B, t) \) for any \( t > \bar{t} \), and \( \epsilon(r^B) < \bar{\epsilon}(r^B, t) \) for any \( t < \bar{t} \). From Proposition 1, \( r^t = r^B \) if \( t \in [\bar{t}, \hat{t}] \) and \( r^t \in \left( \frac{1+i+c}{p}, r^B \right) \) with \( \frac{\partial r^t}{\partial t} < 0 \) for any \( t > \hat{t} \). We can thus set \( \hat{t} = \bar{t} \). If \( \epsilon(r^B) = \bar{\epsilon}(r^B, \bar{t}) \), then \( \frac{\partial \bar{\epsilon}(r^B, \bar{t})}{\partial t} < 0 \) gives us \( \epsilon(r^B) > \bar{\epsilon}(r^B, t) \) for any \( t > \bar{t} \), implying that \( r^t = r^B \) and \( r^t < r^B \) for any \( t > \bar{t} \). Proposition 2 then follows for \( \hat{t} = \bar{t} \). Finally, \( \epsilon(r^B) > \bar{\epsilon}(r^B, \bar{t}) \) implies that \( r^t < r^B \) for any \( t \geq \bar{t} \). We can thus set \( \hat{t} = k \) for any \( k \in [\bar{t}, \bar{t}] \).

**Firm strategy:** Expected profit maximization implies that it is optimal for any type-\( t \) to accept a trade credit offer from an informed supplier if and only if \( r^t \leq r^B \). If this condition is not satisfied or the informed supplier does not offer trade credit, then it is optimal to accept the least costly uninformed loan, which is a bank loan at the interest rate \( r^B \). The first order condition (4) implies that the firm borrows from the bank \( I^*(r^B) \) to buy inputs for the project.

**Part II (Uniqueness):** From Proposition 1, trade credit is strictly profitable for the informed suppliers if the borrower’s type is \( t > \bar{t} \). And trade credit implies an expected loss for the informed suppliers if \( t < \bar{t} \). Hence, it is not optimal for informed suppliers to randomize their trade credit decisions, if the borrower’s type is \( t \neq \bar{t} \). Random strategies for firms and informed suppliers may be optimal, though, if an informed supplier faces the type-\( \bar{t} \) firm. Since the type-\( \bar{t} \) has Lebesgue zero measure in \( [\bar{t}, 1] \), equilibria that differ with respect to strategies involving only the type \( \bar{t} \) are all payoff equivalent, in the sense that they yield the same payoff for each player. We can thus restrict our attention to equilibria in pure strategies.

Consider first a candidate for equilibrium in which informed suppliers never extend trade credit. This strategy leads to no updating upon a request of a bank loan, implying that \( r^B = \frac{1+i}{p E_0(t)} - 1 \), where \( E_0(t) = \int_{\bar{t}}^1 sdF_0(s) \). This bank rate breaks down the candidate because it is strictly profitable for informed suppliers to offer trade credit at \( r^B \) to any \( t > \bar{t} \) (where \( t = \frac{1+i+c}{p(1+r^B)} = \frac{1+i+c}{1+i} E_0(t) < 1 \), with the last inequality following from Assumption 1.
Consider now a candidate for equilibrium in which informed suppliers extend trade credit to all firms. In this proposed equilibrium, bank loans are out of the equilibrium path, implying that any \(E_1[t] \in [t, 1]\) is consistent with Bayes’ rule. The most pessimistic expectation, \(E_1[t] = t\), increases the equilibrium bank rate to its upper bound, \(r^B = \frac{1+i}{1+i} - 1\), thereby maximizing the suppliers’ ability to raise the cost of trade credit. Still, the candidate for equilibrium unravels because it is not profitable for informed suppliers to offer trade credit to any type \(t \in [\frac{t}{1+i}, \frac{t}{1-i}]\), if the cost of trade credit is bounded by at \(r^B = \frac{1+i}{1+i} - 1\).

It then follows that multiple equilibria in pure strategies exist only if there is more than one \(\tilde{t} \in (t, 1)\) that is a root of \(T(t)\). To see that this cannot happen, define \(U(t) = T(t)F_0(t)\) for \(t \in [\frac{t}{1+i}, \frac{t}{1-i}]\). By construction, \(U(t) = 0\) and, for any \(t \neq \tilde{t}\), \(U(t) = 0 \iff T(t) = 0\). Existence of a single root of \(T(t)\) is therefore equivalent to existence of only two roots of \(U(t)\). To show that \(U(t)\) has only two roots, differentiate it to obtain:

\[
U'(t) = F_0(t) + tf_0(t) - \left(1 + \frac{c}{1+i}\right) tf_0(t) = F_0(t) - \frac{c}{1+i} tf_0(t).
\]

If \(t^* \in (\tilde{t}, 1)\) is a critical point of \(U(t)\), then \(U'(t^*) = 0\) and

\[
U''(t^*) = \left(\frac{d}{dt} \left(\frac{F_0(t)}{f_0(t)}\right)\right)|_{t=t^*} - \frac{c}{1+i} f_0(t^*) > 0,
\]

with the strict inequality following from \(\frac{d}{dt} \left(\frac{F_0(t)}{f_0(t)}\right)|_{t=t^*} > \frac{c}{1+i}\). \(U''(t^*) > 0\) implies that every critical point of \(U\) is a local minimum, proving that \(U(t)\) has at most two roots. 

\[\blacksquare\]

**Proof of Proposition 3:** Let \(r^B_k\) be the equilibrium bank rate conditioned on a prior \(k \in \{F_0(t), G_0(t)\}\). From equation (1), \(r^B_k = \frac{1+i}{pE_1^k[t]} - 1\), where \(E_1^k[t]\) is the expected type of a firm that borrows from the bank, conditioned on the prior \(k\). Hence, \(r^B_G > r^B_F\) if and only if \(E_1^G[t] < E_1^F[t]\). To prove that this latter inequality holds, consider \(t \in (\tilde{t}, 1]\) and define

\[
R(t) = \frac{G_0(t)}{F_0(t)} \int_{\tilde{t}}^{t} F_0(s)ds - \int_{\tilde{t}}^{t} G_0(s)ds.
\]

Differentiating \(R(t)\) yields

\[
R'(t) = \frac{d}{dt} \left(\frac{G_0(t)}{F_0(t)}\right) \int_{\tilde{t}}^{t} F_0(s)ds < 0,
\]

35
with the strict inequality following from \( \frac{g_{0}(t)}{\dot{g}_{0}(t)} < \frac{f_{0}(t)}{\dot{f}_{0}(t)} \) for all \( t > \hat{t} \) being equivalent to \( \frac{d}{dt} \left( \frac{G_{0}(t)}{F_{0}(t)} \right) < 0 \) for all \( t > \hat{t} \). Since \( \lim_{t \to \hat{t}} R(t) = 0 \), \( R'(t) < 0 \) implies that \( R(t) < 0 \) for all \( t > \hat{t} \), or equivalently
\[
\frac{1}{G_{0}(t)} \int_{\hat{t}}^{t} G_{0}(s) ds > \frac{1}{F_{0}(t)} \int_{\hat{t}}^{t} F_{0}(s) ds, \text{ for all } t > \hat{t}.
\]

The inequality (27) and integration by parts yield
\[
E_{1}^{G}[s|s \leq \hat{t}] \equiv \frac{1}{G_{0}(t)} \int_{\hat{t}}^{t} g_{0}(s) \ ds = t - \frac{1}{G_{0}(t)} \int_{\hat{t}}^{t} G_{0}(s) ds < t - \frac{1}{F_{0}(t)} \int_{\hat{t}}^{t} F_{0}(s) ds = \frac{1}{F_{0}(t)} \int_{\hat{t}}^{t} f_{0}(s) ds \equiv E_{1}^{F}[s|s \leq \hat{t}],
\]
which implies that
\[
A^{G}(t) \equiv \frac{1 + i + c}{1 + i} E_{1}^{G}[s|s \leq \hat{t}] < \frac{1 + i + c}{1 + i} E_{1}^{F}[s|s \leq \hat{t}] \equiv A^{F}(t).
\]

Let \( \hat{t}_{k} \) be the riskiest type in class \( k \in \{F, G\} \) that has access to trade credit. The proof of Proposition 2 shows that \( \hat{t}_{G} \) is a fixed point of \( A^{G}(t) \) in the interval \( (\hat{t}, 1] \) and \( \hat{t}_{F} \) is a fixed point of \( A^{F}(t) \) in the same interval. Since the inequality (28) shows that \( A^{G}(t) \) lies below \( A^{F}(t) \) for all \( t \in (\hat{t}, 1] \), any fixed point of \( A^{G}(t) \) must lie below \( A^{F}(t) \), that is, \( \hat{t}_{G} < \hat{t}_{F} \). Consequently,
\[
\frac{1 + i + c}{1 + i} E_{1}^{G}[s|s \leq \hat{t}_{G}] = \hat{t}_{G} < \hat{t}_{F} = \frac{1 + i + c}{1 + i} E_{1}^{G}[s|s \leq \hat{t}_{F}] \iff E_{1}^{G}[t] \equiv E_{1}^{G}[s|s \leq \hat{t}_{G}] < E_{1}^{F}[s|s \leq \hat{t}_{F}] = E_{1}^{F}[\hat{t}],
\]
which implies that \( r_{G}^{B} = \frac{1 + i}{p E_{F}[\hat{t}]} - 1 > r_{F}^{B} = \frac{1 + i}{p E_{F}[\hat{t}]} - 1 \).

Since \( r_{G}^{B} > r_{F}^{B} \), \( \hat{t}_{G} \) is smaller than \( \hat{t}_{F} \) if and only if \( \hat{t} \) decreases with \( r^{B} \). To show that \( \hat{t} \) decreases with \( r^{B} \), consider first that no trade credit contract varies with firm characteristics, that is, \( \hat{t} = 1 \), or equivalently, \( \epsilon(r^{B}) \leq \bar{\epsilon}(r^{B}, 1) \). From Assumption 2, the elasticity of demand is non-decreasing in the interest rate, while \( \bar{\epsilon}(r^{B}, 1) \) decreases with \( r^{B} \). Hence, an increase in \( r^{B} \) makes it easier that \( \epsilon(r^{B}) > \bar{\epsilon}(r^{B}, 1) \), implying that \( \hat{t} \) either stays unaltered or falls below one. If \( \epsilon(r^{B}) > \bar{\epsilon}(r^{B}, 1) \), then the safest type that pays \( r^{B} \) (i.e., \( \hat{t} \)) is implicitly defined by \( \epsilon(r^{B}) = \bar{\epsilon}(r^{B}, \hat{t}) \), or equivalently, \( r^{\hat{t}} = r^{B} \), which implies that \( \hat{t} \) strictly decreases with \( r^{B} \), because, from Proposition 2, \( r^{\hat{t}} \) strictly decreases with \( t \) for any \( r^{\hat{t}} < r^{B} \). \[\blacksquare\]

**Proof of Proposition 4:** Plugging \( 1 + r^{B} = \frac{1 + i}{p E_{F}[\hat{t}]} \) into \( \hat{t} = \frac{1 + i + c}{p(1 + r^{B})} \) yields \( \bar{\epsilon}(c) = A(\bar{\epsilon}(c), c) \), where
\[
A(t, c) = \frac{1 + i + c}{1 + i} \frac{1}{F_{0}(t)} \int_{\hat{t}}^{t} s f_{0}(s) ds.
\]
\[\text{(29)}\]
As in the proof of Proposition 2, \( \hat{t}(c) \) is a fixed point of \( A(t, c) \). Since \( A(t, c) \) strictly increases with \( c \), so does its fixed point, proving that \( \hat{t}(c) \) strictly increases with \( c \).

To show that \( \hat{t}(c) \) is strictly increasing in \( c \) and \( \hat{t}(c) - \bar{t}(c) \) is non-decreasing, plug \( 1 + r^B = \frac{1+i}{pE_1[t]} \) and \( \bar{t} = \frac{1+i+c}{1+i} E_1[t] \) into \( \epsilon(r^B(c)) = \epsilon(r^B(c), \hat{t}(c), c) \) to obtain

\[
\epsilon(r^B(c)) = \frac{\hat{t}(c)}{\hat{t}(c) - \bar{t}(c)}, \quad \text{for any } c.
\]  

(30)

We claim that \( \epsilon(r^B(c)) \) cannot increase with \( c \). To see why, write \( 1 + r^B(c) = \frac{1+i}{pE_1[t]} = \frac{1+i}{p} \left( \frac{1}{F_0(t)} \int_{0}^{t} s f_0(s) ds \right)^{-1} \) and note that

\[
\frac{d}{dt} \left( \frac{1}{F_0(t)} \int_{0}^{t} s f_0(s) ds \right) = \frac{tf_0(t)F_0(t) - f_0(t) \int_{0}^{t} s f_0(s) ds}{F_0(t)^2}
\]

(31)

\[
> \frac{tf_0(t)F_0(t) - tf_0(t) \int_{0}^{t} f_0(s) ds}{F_0(t)^2} = 0,
\]

proving that \( r^B(c) \) decreases with \( c \) because we have already established that \( \bar{t}(c) \) increases with \( c \). It then follows that \( \epsilon(r^B(c)) \) cannot increase with \( c \) because, by Assumption 2, \( \epsilon(r) \) is non-decreasing in \( r \). Differentiating equation (30) with respect to \( c \) yields

\[
\frac{de(r^B(c))}{dc} = \frac{\hat{t}(c) \bar{t}'(c) - \bar{t}(c) \hat{t}'(c)}{(\hat{t}(c) - \bar{t}(c))^2},
\]

which implies that \( \frac{de(r^B(c))}{dc} \leq 0 \) if and only if \( \frac{\bar{t}'(c)}{\bar{t}(c)} \geq \frac{\hat{t}'(c)}{\hat{t}(c)} \), with \( \hat{t}'(c) > 0 \) because we have already shown that \( \bar{t}'(c) > 0 \).

\[\blacksquare\]

**Proof of Lemma 1**: Let \( \Pi(B) \) be the value of the objective function (12) given an investment \( B \) in the diversion technology. \( \Pi(B) \) is strictly concave in \( [0, \bar{B}(I)] \), where \( \bar{B}(I) \) is implicitly defined by \( Q(I - \bar{B}(I)) = A(I) \). Hence, \( Q'(I) \geq \rho \) is necessary and sufficient for \( \Pi(0) > \Pi(B) \) for any \( B \in (0, \bar{B}(I)) \). In the interval \( [\bar{B}(I), I] \), \( B = I \) maximizes the objective function because \( \Pi(B) \) strictly increases with \( B \) in \( [\bar{B}(I), I] \). Plugging \( B = I \) and \( B = 0 \) into the objective function of Program (12) yields \( \Pi(0) \geq \Pi(I) \) if and only if \( \frac{Q(I)}{I} \geq \rho + \frac{A(I)}{I} \), proving that \( B(I, \rho) = 0 \) solves Program (12) if \( Q'(I) \geq \rho \) and \( \frac{Q(I)}{I} \geq \rho + \frac{A(I)}{I} \). If \( Q'(I) \geq \rho \) and \( \frac{Q(I)}{I} < \rho + \frac{A(I)}{I} \), then \( \Pi(I) > \Pi(B) \) for any \( B \in [0, I] \), proving that \( B(I, \rho) = I \) is optimal.
Assume now that \( Q'(I) < \rho \). If \( Q'(I - B^*) = \rho \) for some \( B^* \in (0, \bar{B}(I)) \), then \( B^* \) maximizes \( \Pi(B) \) in \([0, \bar{B}(I)]\). Plugging \( B = B^* \) and \( B = I \) into the objective function yields \( \Pi(B^*) \geq \Pi(I) \) if and only if \( \frac{Q(I - B^*)}{I - B^*} \geq \rho + \frac{A(I)}{I - B^*} \). As a result, \( B^* \) solves Program (12) if and only if \( Q'(I - B^*) = \rho \) and \( \frac{Q(I - B^*)}{I - B^*} \geq \rho + \frac{A(I)}{I - B^*} \). If \( Q'(I - B^*) = \rho \) and \( \frac{Q(I - B^*)}{I - B^*} < \rho + \frac{A(I)}{I - B^*} \), then \( B(I, \rho) = I \) is the optimal diversion. Likewise, \( B = I \) is optimal if \( Q'(I - B) < \rho \) for any \( B \in [0, \bar{B}(I)] \), because, in this case, \( \Pi(B) \) strictly increases with \( B \) in \([0, I]\).

\[ \text{Proof of Proposition 5:} \] As in section 2.4, the optimal debt contract maximizes the borrower’s net output while leaving the bank with zero expected profits. Hence, \((I, A(I))\) solves

\[ \max_{(I, A(I))} pE_1[t]\left[ Q(I - B(I, \rho)) + \rho B(I, \rho) \right] - (1 + i)I \tag{32} \]

subject to \( B(I, \rho) = \arg\max_{B \in [0, I]} \left\{ \max \left\{ Q(I - B) - A(I), 0 \right\} + \rho B \right\} \), \( A(I) = \frac{1 + i}{pE_1[t]} I \), \( Q(I - B(I, \rho)) \geq A(I) \). \tag{33} \tag{34} \tag{35}

The objective function (32) is the borrower’s expected output, assuming it invests \( B(I, \rho) \) in the diversion technology. Equation (33) ensures that \( B(I, \rho) \) is optimal for the borrower. Equation (34) implies that the debt contract yields zero-expected profits for the bank, and equation (35) guarantees that the borrower can pay \( A(I) \) in case the project succeeds.

We claim that \( B(I^*, \rho) = 0 \), if \((I^*, A(I^*))\) solves Program (32). To see why, note first that the Inada conditions imply \( I^* > 0 \). Hence, \( B(I^*, \rho) < I^* \) because \( Q(I^* - B(I^*, \rho)) = 0 \) makes it impossible for constraints (34) and (35) to be simultaneously satisfied. Assume, by absurd, that \( B(I^*, \rho) \in (0, I^*) \). From the proof of Lemma 1, the constraint (35) is not binding at \( I^* \), implying that \( Q'(I^* - B(I^*, \rho)) = \rho \) and

\[ Q'(I^* - B(I^*, \rho)) \left( 1 + \frac{\partial B(I^*, \rho)}{\partial I} \right) + \rho \frac{\partial B(I^*, \rho)}{\partial I} = \frac{1 + i}{pE_1[t]}. \tag{36} \]

Applying the Implicit Function Theorem to \( Q'(I^* - B(I^*, \rho)) = \rho \) obtains \( \frac{\partial B(I^*, \rho)}{\partial I} = 1 \). Plugging \( \frac{\partial B(I^*, \rho)}{\partial I} = 1 \) into equation (36) yields \( \rho = \frac{1 + i}{pE_1[t]} \), which cannot hold because, by assumption, \( \rho < \frac{1 + i}{p} \) and \( E_1[t] \leq 1 \).
**Proof of Proposition 6:** Using Lemma 1 and \( A(I) = (1 + r^B)I \), the debt contract \((I, r^B)\) induces the borrower to invest the full amount of the loan in the project if and only if

\[
Q'(I) \geq \rho \quad \text{and} \quad \frac{Q(I)}{I} \geq \rho + (1 + r^B).
\]

The Inada conditions imply that \( \lim_{t \to 0} \frac{Q(I)}{I} = Q'(0) = \infty \) and \( \lim_{t \to \infty} \frac{Q(I)}{I} = Q'(\infty) = 0. \)

Since \( \frac{Q(I)}{I} \) decreases with \( I \), continuity of \( \Psi(I) = \frac{Q(I)}{I} - \rho - (1 + r^B) \) implies that there is \( \bar{I}(r^B, \rho) \) such that \( \frac{Q(I(r^B, \rho))}{I(r^B, \rho)} = \rho + (1 + r^B) \), with \( \frac{Q(I)}{I} \geq \rho + (1 + r^B) \) if and only if \( I \leq \bar{I}(r^B, \rho) \). Hence, \((I, r^B)\) satisfies condition (38) if and only if \( I \leq \bar{I}(r^B, \rho) \), and concavity of \( Q(I) \) implies that \( \bar{I}(r^B, \rho) \) strictly decreases with \( \rho \) and \( r^B \).

If \( I^*(r^B) \leq \bar{I}(r^B, \rho) \), then Lemma 1 implies that \((I^*(r^B), r^B)\) induces the borrower to invest \( I^*(r^B) \) in the project if and only if \( Q'(I^*(r^B)) \geq \rho \). To see that this condition holds, recall that the first order condition that characterizes the unconstrained optimal investment \( I^*(r^B) \) is \( Q'(I^*(r^B)) = 1 + r^B \). Plugging the equilibrium bank rate into \( 1 + r^B \) yields \( Q'(I^*(r^B)) = \frac{1 + i}{\rho E_1[t]} \geq \frac{1 + i}{\rho} \) \( > \rho \), with the weak inequality from \( E_1[t] \leq 1 \) and the strict inequality from \( \rho < \frac{1 + i}{\rho} \).

It then follows that \( I^*(r^B) \) satisfies condition (37), proving that \((I^*(r^B), r^B = \frac{1 + i}{\rho E_1[t]} - 1)\) is the unique equilibrium contract, because it rules out agency costs of debt, yields zero-expected profits for the bank, and finances the optimal investment in the project.

If \( \bar{I}(r^B, \rho) < I^*(r^B) \), then \((I^*(r^B), r^B)\) does not satisfy the incentive condition (38). From Lemma 1 and Proposition 5, the equilibrium debt contract – call it \((I^B, r^B)\) – must satisfy \( I^B \leq \bar{I}(r^B, \rho) < I^*(r^B) \). Since \( Q'(I^*(r^B)) \geq \rho \), concavity of \( Q(I) \) implies that \( Q'(I^B) > \rho \), proving that any \( I^B \leq \bar{I}(r^B, \rho) \) satisfies conditions (38) and (37). Concavity of the production function implies that it is optimal for the firm to keep the scale of the project as close as possible to the unconstrained optimum. Hence, the equilibrium contract is \((\bar{I}(r^B, \rho), r^B)\).

**Proof of Proposition 7:** Assume first that \( \rho = 0 \). In this case, the shareholders’ gain of diverting cash is zero, making it optimal for any firm to invest all available funds in the project. From Proposition 6, \((I^*(r^B), r^B = \frac{1 + i}{\rho E_1[t]} - 1)\) is the equilibrium debt contract in the market for bank loans, fetching zero expected profits for the bank, while financing the optimal
scale of the project. Conversely, it is optimal for any firm to accept \((I^*(r^B), r^B = \frac{1+i}{rE_1[t]} - 1)\), whenever it has no access to trade credit, or the cost of trade credit is larger than \(r^B\). The largest cost of trade credit that firms are willing to pay is thus \(\bar{r} = r^B\).

As in Proposition 2, we look for an equilibrium in which the informed suppliers offer trade credit to any \(t \geq \bar{t} \in (\underline{t}, 1)\). In this equilibrium, the type-\(t\) firm borrows from the bank if and only if \(t < \bar{t}\). Bayes’ rule thus implies that \(F_1(t) = F_0(t)/F_0(\bar{t})\), \(E_1[t] = \frac{1}{F_0(\bar{t})} \int_{\underline{t}}^{\bar{t}} s f_0(s) ds\), and

\[
1 + r^B = \frac{1+i}{\frac{p}{F_0(\bar{t})} \int_{\underline{t}}^{\bar{t}} s f_0(s) ds}.
\]

Plugging \(\bar{r} = r^B\) into the right-hand side of inequality (18) yields

\[
\bar{t} = \frac{1+i+c}{\frac{p}{F_0(\bar{t})} \int_{\underline{t}}^{\bar{t}} s f_0(s) ds}.
\]  

Following the same arguments used in the proof of Proposition 2, one can check that there exists \(\bar{t}(0) \in (\underline{t}, 1)\) that satisfies equation (39). The cutoff type \(\bar{t}(0)\) pins down the equilibrium bank rate, \(1 + r^B(0) = \frac{1+i}{\frac{p}{F_0(\bar{t}(0))} \int_{\underline{t}}^{\bar{t}(0)} s f_0(s) ds}\), and the maximum interest rate that the informed supplier can charge, \(\bar{r}(0) = r^B(0)\). To complete the characterization of the equilibrium, the elasticity condition, \(\epsilon(r^B(0)) = \bar{\epsilon}(r^B(0), \bar{t}(0))\), yields the safest type \(\hat{t}(0)\) that pays \(r^B(0)\) to the informed supplier. As in the proof of Proposition 2, \(r^t(0) = r^B(0)\) for any \(t \leq \hat{t}(0)\). If \(t > \hat{t}(0)\), then the optimal interest rate is implicitly defined by \(\epsilon(r^t(0)) = \bar{\epsilon}(r^t(0), t)\), with \(r^t(0) \in \left(\frac{1+i+c}{p}, r^B(0)\right)\). Denote this equilibrium by

\[
\mathcal{E}(0) = \left\{ \left( I^*(r^B(0)), r^B(0) \right), \left( I^*(r^t), r^t \right)_{t \in [\bar{t}(0), 1]} \left\{ 1^t_{\text{bank}} \right\}_{t \in [\underline{t}, 1]}, F_1(t) = F_0(t)/F_0(\bar{t}(0)) \right\}.
\]

If \(\frac{d}{dt} \left( \frac{F_0(t)}{F_0(\bar{t})} \right) > \frac{c}{1+i}\), then the proof of uniqueness in Proposition 2 applies to \(\mathcal{E}(0)\). Indeed, the arguments in the proof of Proposition 2 imply that \(\mathcal{E}(0)\) is the unique equilibrium for any \(\rho\) that assures that firms invest in the project after accepting a bank offer that does not impose credit constraint. Our next task, therefore, is to find the values of \(\rho\) that let the bank offer \((I^*(r^B(0)), r^B(0))\), without inducing agency costs of debt. For any \(\rho > 0\), define

\[
\mathcal{E}(0, \rho) = \left\{ \left( \bar{I}^B(r^B(0), \rho), r^B(0) \right), \left( I^*(r^t), r^t \right)_{t \in [\bar{t}(0), 1]} \left\{ 1^t_{\text{bank}} \right\}_{t \in [\underline{t}, 1]}, F_0(t)/F_0(\bar{t}(0)) \right\},
\]

where \(\bar{I}^B(r^B(0), \rho)\) is the investment defined in equation (14), with \(r^B = r^B(0)\).

We demonstrate below that \(\mathcal{E}(0) = \mathcal{E}(0, \rho)\) if and only if \(\rho = \rho^{\text{const}}\), where \(\rho^{\text{const}} > 0\) is the unique value of \(\rho\) such that \(\bar{I}^B(r^B(0), \rho) = I^*(r^B(0))\). The characterization of \(\rho^{\text{const}}\) proves that
\(E(0)\) is the unique equilibrium for any \(\rho \leq \rho^{\text{const}}\), because, for these values of \(\rho\), \(\overline{I}^B(r_B(0), \rho) \geq I^*(r_B(0))\), which, from Proposition 6, is a sufficient condition for \((I^*(r_B(0)), r_B(0))\) to be an equilibrium debt contract that does not imply credit constraint.

To obtain \(\overline{I}^B(r_B(0), \rho^{\text{const}}) = I^*(r_B(0))\), fix \(r_B(0)\) and define \(\Psi(I^B, r_B(0), \rho) = \frac{Q(I^B)}{I^B} - \rho - (1 + r_B(0)).\) It then follows that \(\Psi(\overline{I}^B(r_B(0), \rho), r_B(0), \rho) = 0\) and

\[
\lim_{\rho \to \infty} \Psi(\overline{I}^B(r_B(0), \rho), r_B(0), \rho) = 0 \Rightarrow \lim_{\rho \to \infty} \frac{Q(\overline{I}^B(r_B(0), \rho))}{I^B(r_B(0), \rho)} = \infty \Rightarrow \lim_{\rho \to \infty} \overline{I}^B(r_B(0), \rho) = 0,
\]

with the second implication following from the Inada conditions. Analogously, \(\lim_{\rho \to 0} \frac{Q(\overline{I}^B(r_B(0), \rho))}{I^B(r_B(0), \rho)} = 1 + r_B(0) = Q'(I^*(r_B(0)))\), with the second equality following from the first order condition (4).

Concavity of the production function and the Inada conditions imply that \(\lim_{\rho \to 0} \frac{Q(\overline{I}^B(r_B(0), \rho))}{I^B(r_B(0), \rho)} = Q'(I^*(r_B(0)))\) only if \(\lim_{\rho \to 0} \overline{I}^B(r_B(0), \rho) = \infty\). Hence, \(\overline{I}^B(r_B(0), \rho)\) is arbitrarily large when \(\rho\) is close to 0 and decreases to 0 when \(\rho\) goes to infinity. From Proposition 6, \(\overline{I}^B(r_B(0), \rho)\) strictly decreases with \(\rho\). As a result, there exists \(\rho^{\text{const}} > 0\) such that \(\overline{I}^B(r_B(0), \rho^{\text{const}}) = I^*(r_B(0))\), with \(\overline{I}^B(r_B(0), \rho) > I^*(r_B(0))\) for any \(\rho < \rho^{\text{const}}\) and \(\overline{I}^B(r_B(0), \rho) < I^*(r_B(0))\) for any \(\rho > \rho^{\text{const}}\), proving that \(E(0)\) is the unique equilibrium for any \(\rho \leq \rho^{\text{const}}\) with \(E(0) = E(0, \rho^{\text{const}})\).

Finally, assume, by absurd, that there exists an equilibrium without credit constraint for some \(\rho > \rho^{\text{const}}\). This equilibrium must be \(E(0)\), because the arguments in the proof of Proposition 2 prove that there is a unique equilibrium without credit constraint that induces the borrowing firms to invest in the project. A contradiction then follows because \(\rho > \rho^{\text{const}}\) if and only if \(\overline{I}^B(r_B(0), \rho) < I^*(r_B(0))\), which implies (see Proposition 6) that \((I^*(r_B(0)), r_B(0))\) does not induce the borrowing firms to invest in the project. \(\blacksquare\)

**Proof of Proposition 8:** The proof is divided in two parts. The first part proves existence of equilibrium. The second part exhibits sufficient conditions for the equilibrium to be unique.

*Part I. Existence of equilibrium:* For any \(\rho \in (\rho^{\text{const}}, \frac{1+i}{p})\), we claim that the strategies and belief represented by \(E(\rho)\) (see equation (19)) form a Perfect Bayesian Equilibrium if and only
\[ \xi(\rho) = \left( r^B(\rho), \bar{r}(\rho), \bar{t}(\rho) \right) \] satisfies the following conditions:

\[ p(1 + \bar{r}(\rho))\bar{t}(\rho) - (1 + i + c) \geq 0 \tag{40} \]

\[ Q(I^*(\bar{r}(\rho))) - (1 + \bar{r}(\rho))I^*(\bar{r}(\rho)) - \left( Q(\bar{I}^B(r^B(\rho), \rho)) - (1 + r^B(\rho))\bar{I}^B(r^B(\rho), \rho) \right) = 0 \tag{41} \]

\[ p(1 + r^B(\rho))E_1[t|\bar{t}(\rho)] - (1 + i) = 0 \tag{42} \]

\[ E_1[t|\bar{t}(\rho)] = \begin{cases} 
\frac{1}{F_0(\bar{t}(\rho))} \int_{\bar{t}}^{\rho} s f_0(s)ds & \text{if } \bar{t}(\rho) > \tilde{t}, \\
E_1[t] & \text{if } \bar{t}(\rho) = \tilde{t}.
\end{cases} \tag{43} \]

Inequality (40) is a necessary condition for \( \bar{t}(\rho) \) to be the cutoff type that has access to trade credit, given the largest interest rate \( \bar{r}(\rho) \) that firms are willing to pay to the informed suppliers. The inequality holds strictly, if trade credit is strictly profitable for any \( t \geq \tilde{t} \). Equation (41), which is equivalent to equation (16), determines \( \bar{r}(\rho) \). Equation (42) is the zero-profit condition for the bank, and (43) is the condition for \( E_1[t|\bar{t}(\rho)] \) to be the expected type of a firm that borrows from the bank. If bank loans are in the equilibrium path, i.e. \( \bar{t}(\rho) > \tilde{t} \), then Bayes’ rule implies that \( E_1[t|\bar{t}(\rho)] = \frac{1}{F_0(\bar{t}(\rho))} \int_{\bar{t}}^{\rho} s f_0(s)ds \). If \( \bar{t}(\rho) = \tilde{t} \), Bayes’ rule is consistent with any \( E_1[t|\bar{t}(\rho)] \in [\tilde{t}, 1] \).

To prove that any \( \xi(\rho) \) that satisfies conditions (40) to (43) makes \( E(\rho) \) a PBE, note first that, by construction of \( I^B(r^B(\rho), \rho) \), the debt contract \( \left( \bar{I}^B(r^B(\rho), \rho), r^B(\rho) \right) \) dissuades the borrower from diverting cash, while keeping the scale of the project as close as possible to the first best. Since \( r^B(\rho) \) satisfies equation (42) and there are no incentives to divert cash, the debt contract yields zero expected profits, proving that \( \left( \bar{I}^B(r^B(\rho), \rho), r^B(\rho) \right) \) is optimal for the bank. To see that the trade credit contracts \( \left( I^*(r^t(\rho)), r^t(\rho) \right)_{t \in [\tilde{t}, 1]} \) are optimal for the informed suppliers, note that equation (41) implies that \( \bar{r}(\rho) \) is the largest interest rate that firms are willing to pay given the outside option \( \left( \bar{I}^B(r^B(\rho), \rho), r^B(\rho) \right) \). Hence, condition (40), which takes into account \( \bar{r}(\rho) \), implies that it is profitable for the informed suppliers to extend trade credit to any \( t \geq \bar{t}(\rho) \). Optimality of \( \left( I^*(r^t(\rho)), r^t(\rho) \right)_{t \in [\tilde{t}, 1]} \) thus follows because, by construction of \( E(\rho) \), \( r^t(\rho) \) solves program (17). Finally, condition (43) ensures that the belief \( F_1(t|\bar{t}(\rho)) \) satisfies Bayes’ rule whenever possible, proving that \( E(\rho) \) is a PBE.

Conversely, lack of a solution to conditions (40) to (43) rules out the existence of strategies that maximize expected payoffs given the other players’ strategies and a belief \( F_1(t|\bar{t}(\rho)) \) that is
consistent with Bayes’ rule, whenever possible. Existence of equilibrium is therefore equivalent to existence of $\xi(\rho)$ that satisfies conditions (40) to (43).

We want to prove that, for any $\rho \in (\rho^{const}, \frac{1 + \bar{t}}{p})$, there exists $\xi(\rho)$ that satisfies conditions (40) to (43). As a first step towards this proof, recall that Proposition 7 shows that $E(\rho^{const}) = E(0)$, with $\bar{t}(\rho^{const}) = \bar{t}(0) \in (\bar{t}, 1)$. Hence, $\xi(0) = \xi(\rho^{const})$ and the inequality (40) is binding at $\xi(\rho^{const})$, implying that $E_1[t|\tilde{t}(\rho^{const})] = \frac{1}{F_0(\tilde{t}(\rho^{const}))} \int_\tilde{t}^{\bar{t}(\rho^{const})} s f_0(s) \, ds$. If $\rho \in (\rho^{const}, \rho^{const} + \epsilon)$ with $\epsilon > 0$ small enough, then the Implicit Function Theorem assures that there is a unique $\xi(\rho)$ that solves equations (40), (41), and (42) if and only if

$$\Delta(\rho^{const}) = \left(1 + \bar{t}ight) I^*(\bar{t}) E_1[t|\bar{t}] - \bar{t}(1 + rB) \frac{dE_1[t|\bar{t}]}{dt} \left(Q'((\bar{t}) - (1 + rB) \frac{\partial B}{\partial \rho} \bar{I}_B \bar{I}_B \right) > 0,$$ \hspace{1cm} (44)

where $(\bar{I}_B, r^B, \bar{t}, E_1[t|\bar{t}] = (\bar{I}_B(r^B(\rho^{const}), \rho^{const}), r^B(\rho^{const}), \bar{I}(\rho^{const}), \bar{t}(\rho^{const}), E_1[t|\tilde{t}(\rho^{const})])$.

To evaluate the sign of $\Delta(\rho^{const})$, check in the proof of Proposition 4 (see equation (31)) that $\frac{dE_1[t|\bar{t}]}{dt} > 0$, whenever $\tilde{t} > \bar{t}$. Moreover, Proposition 6 and the first order condition of the firm’s investment problem imply that $Q'((\bar{I}_B) - (1 + rB) \frac{\partial B}{\partial \rho} \bar{I}_B \bar{I}_B \leq 0$, with strict inequality if and only if there is credit constraint, that is, $\bar{I}_B < I^*(rB(\rho^{const}))$. Hence, $\Delta(\rho^{const}) > 0$, proving that, for $\epsilon$ close to zero and $\rho \in (\rho^{const}, \rho^{const} + \epsilon)$, there exists $\xi(\rho)$ that solves conditions (40) to (43) with $E_1[t|\tilde{t}(\rho)] = \frac{1}{F_0(\tilde{t}(\rho))} \int_\tilde{t}^{\bar{t}(\rho)} s f_0(s) \, ds$.

To see how $r^B(\rho), \bar{t}(\rho)$ and $\tilde{t}(\rho)$ change with $\rho$ around $\rho^{const}$, differentiate the system of equations (40), (41) and (42) to obtain

$$\begin{bmatrix}
\frac{d\rho}{dp}
\frac{dr^B(\rho)}{dp}
\frac{d\bar{t}(\rho)}{dp}
\end{bmatrix}
= - \left( p^2 \right) \frac{(Q'((\bar{I}_B) - (1 + rB) \frac{\partial B}{\partial \rho} \bar{I}_B \bar{I}_B \Delta(\rho^{const}))}{\bar{t}(1 + rB) \frac{d}{dt} E_1[t|\bar{t}]}
\begin{bmatrix}
(1 + \bar{t}) E_{1}[t|\bar{t}]
\bar{t}(1 + rB) \frac{d}{dt} E_1[t|\bar{t}]
-\bar{t} E_1[t|\bar{t}]
\end{bmatrix},$$

which implies $\frac{d\rho}{dp} \geq 0$, $\frac{dr^B(\rho)}{dp} \geq 0$ and $\frac{d\bar{t}(\rho)}{dp} \leq 0$, with equality if and only if there is no credit constraint, that is, $Q'((\bar{I}_B) - (1 + rB) = 0$. Differentiating twice $\bar{t}(\rho), r^B(\rho)$ and $\tilde{t}(\rho)$ shows that their second derivatives at $\rho^{const}$ are all different from zero:

$$\frac{d^2 \bar{t}}{d\rho^2}(\rho^{const}) = - \left( p^2 \right) Q''((\bar{I}_B) \frac{\partial B}{\partial \rho} \bar{I}_B \bar{I}_B \Delta(\rho^{const})) > 0,$$

$$\frac{d^2 r^B}{d\rho^2}(\rho^{const}) = - \left( p^2 \right) Q''((\bar{I}_B) \frac{\partial B}{\partial \rho} \bar{I}_B \bar{I}_B \Delta(\rho^{const})) > 0,$$

$$\frac{d^2 \tilde{t}}{d\rho^2}(\rho^{const}) = \left( p^2 \right) Q''((\bar{I}_B) \frac{\partial B}{\partial \rho} \bar{I}_B \bar{I}_B \Delta(\rho^{const}) < 0.$$
which proves that $\bar{r}(\rho) > \bar{r}(\rho^\text{const})$, $r^B(\rho) > r^B(\rho^\text{const})$ and $\bar{t}(\rho) < \bar{t}(\rho^\text{const})$, for any $\rho$ slightly larger than $\rho^\text{const}$. From Lemma 2, credit constraint in the market for bank loans is a necessary condition for $\bar{r}(\rho) > \bar{r}(\rho^\text{const}) = r^B(\rho^\text{const})$. As a result, $\bar{I}^B(r^B(\rho), \rho) < \star^+(r^B(\rho))$, for any $\rho \in (\rho^\text{const}, \rho^\text{const} + \varepsilon)$, implying that $\frac{d\bar{r}(\rho)}{d\rho} \geq 0$, $\frac{dr^B(\rho)}{d\rho} \geq 0$, and $\frac{d\bar{t}(\rho)}{d\rho} \leq 0$, with equality only at $\rho^\text{const}$. To show that there exists $\xi(\rho)$ for any $\rho \in \left[ \rho^\text{const}, \frac{1+i}{p} \right)$, define

$$\Theta = \left\{ \tilde{\rho} \in \left[ \rho^\text{const}, \frac{1+i}{p} \right) : \xi(\rho) \text{satisfies (40) to (43) and } \xi_k(\rho) \text{ is weakly monotone for any } k \text{ and } \rho \in [\rho^\text{const}, \tilde{\rho}) \right\},$$

where $\xi_k(\rho)$ is the projection of $\xi(\rho)$ on its component $k \in \{r^B(\rho), \bar{r}(\rho), \bar{t}(\rho)\}$.

Since $\Theta$ is a non empty subset of $\left[ \rho^\text{const}, \frac{1+i}{p} \right)$, there exists $\bar{\rho} = \sup \Theta$. By definition of the sup, there is an increasing sequence $(\rho^n)$ that converges to $\bar{\rho}$ with $\rho^n \in \Theta$ for every $n$. Notice that $r^B(\rho^n) \leq \frac{1+i}{p} - 1$, $\bar{r}(\rho^n) \leq \frac{\rho^n(\star^+(\frac{1+i}{p} - 1))-(1+i)\star^+(1+(\frac{1+i}{p} - 1))}{\star^+(1+(\frac{1+i}{p} - 1))} - 1$, $\bar{t}(\rho^n) \leq 1$. And, by monotonicity of $\bar{r}(\cdot)$, $r^B(\cdot)$ and $\bar{t}(\cdot)$ in $[\rho^\text{const}, \rho^n]$, there exists a unique limit point of $\xi(\rho^n)$, which we denote $\xi(\bar{\rho})$. The limit point $\xi(\bar{\rho})$ satisfies the conditions (40) to (43) and has weakly monotonic projections, implying that $\bar{\rho} \in \Theta$.

Assume, by absurd, that $\bar{\rho} < \frac{1+i}{p}$. There are two cases to analyze: $\bar{t}(\bar{\rho}) > \bar{t}$ and $\bar{t}(\bar{\rho}) = \bar{t}$.

If $\bar{t}(\bar{\rho}) > \bar{t}$, then the constraint (40) is binding and the Implicit Function Theorem assures that there is a solution to the system of equations (40) to (43) in a neighborhood of $\bar{\rho}$, provided that $\Delta(\bar{\rho}) > 0$, where $\Delta(\bar{\rho})$ is defined as in equation (44), with $(\bar{I}^B, r^B, \bar{r}, \bar{t}, E_1[t|\bar{t}]) \equiv (\bar{I}^B(r^B(\bar{\rho}), \bar{\rho}), r^B(\bar{\rho}), \bar{r}(\bar{\rho}), \bar{t}(\bar{\rho}), E_1[t|\bar{t}(\bar{\rho})])$. The same arguments that prove that $\Delta(\rho^\text{const}) > 0$ also imply $\Delta(\rho) > 0$. Moreover, the Implicit Function Theorem implies that $\bar{r}(\cdot), r^B(\cdot)$ and $\bar{t}(\cdot)$ are monotonic functions in a neighborhood of $\bar{\rho}$. But then, there is $\tilde{\rho} \in \Theta$ such that $\bar{\rho} > \tilde{\rho}$, which contradicts $\bar{\rho}$ picked as the sup.

Consider now $\bar{t}(\bar{\rho}) = \bar{t}$ with $\xi(\bar{\rho}) \equiv \left( r^B(\bar{\rho}) = \frac{1+i}{pE_1[\bar{t}]} - 1, \bar{r}(\bar{\rho}) = \bar{r}(\bar{\rho}, E_1[\bar{t}]), \bar{t}(\bar{\rho}) = \bar{t} \right)$, where $\bar{r}(\bar{\rho}, E_1[\bar{t}])$ solves equation (41) given $r^B(\bar{\rho}) = \frac{1+i}{pE_1[\bar{t}]} - 1$, for some $E_1[\bar{t}] \in [\bar{t}, 1]$. If $E_1[\bar{t}] > \bar{t}$, then $\left( \frac{1+i}{p} - 1, \bar{r}(\bar{\rho}, \bar{t}), \bar{t} \right)$ also satisfies (40) to (43), provided that we choose $\bar{r}(\bar{\rho}, \bar{t})$ to solve equation (41) given $r^B = \frac{1+i}{p} - 1$; $\bar{r}(\bar{\rho}, \bar{t})$ exists because, for any $r^B$, $\bar{I}^B(r^B, \rho)$ decreases with $\rho$, mapping values of $\rho$ into $[0, \infty]$. We can thus assume that $E_1[\bar{t}] = \bar{t}$.

We claim that any $\rho \in \left( \bar{\rho}, \frac{1+i}{p} \right)$ belongs to $\Theta$. To see this, recall, from Proposition 6, that $\bar{I}^B\left( \frac{1+i}{pE_1[\bar{t}]} - 1, \bar{\rho} \right)$ decreases as $\bar{\rho}$ increases to $\rho$. The stricter credit constraint increases the largest interest rate that informed suppliers can charge to $\bar{r}(\rho, \bar{t}) > \bar{r}(\bar{\rho}, \bar{t})$, where $\bar{r}(\rho, \bar{t})$ solves equation (41) given $r^B(\rho) = \frac{1+i}{p} - 1$. Despite $\bar{r}(\rho, \bar{t}) > \bar{r}(\bar{\rho}, \bar{t})$, trade credit does not become
more pervasive because the informed suppliers were already offering trade credit to all firms at $\tilde{t}$. Hence, $\tilde{t} = \underline{t}$ is still the cutoff type for access to trade credit, making it optimal for the bank to offer credit at the interest rate $r^B(\rho) = \frac{1+i}{pE_1[\rho]} - 1$, with leeway for us to choose $E_1[\tilde{t}] = \tilde{t}$ because bank loans are off the equilibrium path. This shows that $\xi(\rho)$ satisfies conditions (40) to (43). By construction, $r^B(\rho)$ and $\tilde{t}(\rho)$ do not vary in the interval $\left(\rho, \frac{1+i}{p}\right)$, while equation (41) implies that $\tilde{r}(\rho)$ is monotonic once we take into account that $r^B(\rho) = \frac{1+i}{p} - 1$. It then follows that $\rho \in \Theta$ for any $\rho \in \left(\rho, \frac{1+i}{p}\right)$, contradicting $\tilde{\rho}$ as the sup of $\Theta$. We conclude that $\tilde{\rho} = \frac{1+i}{p}$, proving that $E(\rho)$ is a PBE for any $\rho \in \left(\rho^{const}, \frac{1+i}{p}\right)$.

**Part II. Uniqueness of Equilibrium:** The proof of Proposition 2 rules out mixed-strategy equilibria, whether there is credit constraint or not. Ignoring credit constraint, the proof of Proposition 2 also shows that there is no equilibrium without trade credit. As a result, credit constraint cannot support equilibria without trade credit, because weaker competition from the bank strengthens the incentives for the informed suppliers to offer trade credit. Hence, we can restrict our attention to two types of equilibrium: Equilibria in which all firms have access to trade credit and equilibria in which some types do not have access to trade credit.

From Part-I, there exists $\xi_i(\rho) = \left(r^B_i(\rho), \tilde{r}_i(\rho), \tilde{t}_i(\rho)\right)$ that satisfies conditions (40) to (43). Given $\xi_i(\rho)$, define

$$\rho_{i}^{\text{all}} = \inf \left\{ \rho \in \left(\rho^{\text{const}}, \frac{1+i}{p}\right) : \tilde{t}_i(\rho) = \tilde{t} \right\},$$

with the understanding that $\rho_{i}^{\text{all}} = \frac{1+i}{p}$ if $\tilde{t}_i(\rho) \neq \tilde{t}$ for any $\rho \in \left(\rho^{\text{const}}, \frac{1+i}{p}\right)$. If $\rho < \min\{\rho_{i}^{\text{all}}, \frac{1+i}{p}\}$, Part-I showed that $\frac{\partial r_i(\rho)}{\partial \rho} > 0$, $\frac{\partial r^B_i(\rho)}{\partial \rho} > 0$ and $\frac{\partial \tilde{t}_i(\rho)}{\partial \rho} < 0$, implying that $\tilde{t}_i(\rho) > \tilde{t}$ for any $\rho < \rho_{i}^{\text{all}}$. Denote by $E_i(\rho)$ the PBE induced by $\xi_i(\rho)$. Suppose, by absurd, that there exists $\rho < \rho_{i}^{\text{all}}$ and an equilibrium $E_j(\rho) \neq E_i(\rho)$ that denies trade credit to some firms. From Part-I, $E_j(\rho)$ must be tied to some $\xi_j(\rho)$ that solves equations (40) to (43). The same arguments that prove that $\Delta(\rho^{\text{const}}) > 0$ imply $\Delta(\rho) > 0$ for any $\rho > \rho^{\text{const}}$ such that $\min\{\tilde{t}_i(\rho), \tilde{t}_j(\rho)\} > \tilde{t}$. Each extension of $E_i(\rho)$ and $E_j(\rho)$ is therefore unique, implying that they fold back to different equilibria at $\rho^{\text{const}}$. But this is not possible because Proposition 7 shows that $E(\rho^{\text{const}})$ is the unique equilibrium for $\rho = \rho^{\text{const}}$. Hence, $E_i(\rho) = E_j(\rho)$ for any $\rho < \rho_{i}^{\text{all}}$.

Consider now equilibria in which the informed suppliers offer trade credit to all firms.
In this equilibrium, bank loans are zero probability events. As a result, Bayes’ rule does not restrict the updated belief $F_1(t | \bar{\ell}(\rho))$. Still, we claim that the consistency requirement of Kreps and Wilson (1982) implies $\text{Prob}(s = t | t < \bar{\ell}(\rho)) = 1$. To see why, consider a sequence of shocks that induce some types to request a bank loan with positive probability. Each shock defines a bank rate $r^B(F^n_1) = \frac{1}{\int_{t}^{\bar{\rho}} \text{d}F^n_1(t)} - 1$, where $F^n_1(t)$ is the Bayesian-updated belief under the $n^{th}$-shock. Given $r^B(F^n_1)$, equation (41) yields the maximum cost of trade credit, $\bar{r}(F^n_1)$, which, in turn, makes it strictly profitable for the informed suppliers to offer trade credit to any type $t > t^n \equiv \bar{\ell}(\rho, F^n_1) = \frac{1+i+c}{p(1+i+r(F^n_1))}$. Hence, $t^n$ is the safest type that may ask for a bank loan, and $\lim_{n \to \infty} t^n = t$. For any type $t$, the updated distribution $F^n_1(t)$ is given by

$$F^n_1(t) = \frac{F_0(t)}{F_0(t^n)} 1_{[t<t^n]} + 1_{[t\geq t^n]}, \quad (46)$$

where $1_{[t<z]}$ is the indicator function that takes value one if $t < z$ and zero otherwise.

Taking limits in equation (46) yields $\lim_{n \to \infty} F^n_1(t) = 1$ for any $t$, proving that $\text{Prob}(t = t | t < \bar{\ell}(\rho)) = 1$ is the only consistent belief. This belief implies that $E_1[t] = t$, establishing that $\xi(\rho) = \left( r^B(\rho) = \frac{1+i}{p}, -1, \bar{r}(\rho, t), \bar{\ell}(\rho) = t \right)$ is the unique candidate for delivering a sequential equilibrium without bank loans.

To exhibit conditions for $\xi(\rho)$ to be a valid candidate, define $\rho^{all}$ as in equation (45). If $\rho^{all} = \frac{1+i}{p}$, then there is no sequential equilibrium in which the informed suppliers offer trade credit to all types. In contrast, $\rho^{all} < \frac{1+i}{p}$ implies – from Part-I – that there exists $\xi(\rho)$ that satisfies conditions (40) to (43) with $\bar{\ell}(\rho) > t$ for any $\rho \in (\rho^{\text{const}}, \rho^{\text{all}})$ and $\bar{\ell}(\rho) = t$ for any $\rho \in [\rho^{all}, \frac{1+i}{p})$. For these values of $\rho$, $\xi(\rho)$ is a valid candidate for a sequential equilibrium without bank loans because it satisfies (41) and (42), while taking as given $\bar{\ell}(\rho) = t$, $r^B(\rho) = \frac{1+i}{p} - 1$ and $E_1[t | \bar{\ell}(\rho)] = t$. Conversely, $\rho < \rho^{all}$ rules out $\xi(\rho)$ as a valid candidate. To see why, recall that a necessary condition for $\xi(\rho)$ to make bank loans out-of-equilibrium events is $t \geq \frac{1+i+c}{p(1+r(\rho))} = \bar{\ell}(\rho)$. By construction of $\rho^{all}$, $t = \bar{\ell}(\rho^{all})$. Since $\frac{d\bar{\ell}(\rho)}{dp} \leq 0$, with strict inequality for any $\rho \in (\rho^{\text{const}}, \rho^{\text{const}} + \epsilon)$ and $\epsilon > 0$ sufficiently small, we must have $t < \bar{\ell}(\rho)$ for any $\rho < \rho^{all}$, breaking down the candidate for equilibrium without bank loans. 

**Proof of Proposition 9:** Given $\rho$, the necessary and sufficient condition for $r^t(\rho)$ to be optimal, $\epsilon(r^t(\rho)) \leq \bar{\ell}(r^t(\rho), t)$, implies that no trade credit contract varies with firm characteristics.
if $\epsilon(\bar{r}(\rho)) \leq \bar{\epsilon}(\bar{r}(\rho), 1)$. If this condition holds for any $\rho$, then $\bar{\rho} = \frac{1+i}{p}$ is the cutoff value for $\rho$, below which the cost of trade credit never varies with firm characteristics. Assume now that $\epsilon(\bar{r}(\rho)) > \bar{\epsilon}(\bar{r}(\rho), 1)$, for any $\rho \geq 0$. In this case, there is always a trade credit contract whose interest rate increases with firm-risk factors. We can thus set $\bar{\rho} = k$ for any $k < 0$. Finally, assume that $\epsilon(\bar{r}(\hat{\rho})) = \bar{\epsilon}(\bar{r}(\hat{\rho}), 1)$ for some $\hat{\rho} > 0$. From Assumption 2, the elasticity $\epsilon(r)$ is non-decreasing in the interest rate $r$ and, by construction, $\bar{\epsilon}(\bar{r}(\rho), 1)$ decreases with $\bar{r}(\rho)$, which, in turn, is a non-decreasing function of $\rho$ (see Proposition 8). As a result, $\epsilon(\bar{r}(\hat{\rho})) = \bar{\epsilon}(\bar{r}(\hat{\rho}), 1)$ at $\hat{\rho}$ only, with $\epsilon(\bar{r}(\rho)) < \bar{\epsilon}(\bar{r}(\rho), 1)$ for any $\rho < \hat{\rho}$. The cutoff value for $\rho$ is therefore $\bar{\rho} = \hat{\rho}$.

To show that $\frac{di(\rho)}{dp} \leq 0$, with strict inequality whenever $\frac{di(\rho)}{dp} > 0$, apply the Implicit Function Theorem to $\epsilon(\bar{r}(\rho)) = \bar{\epsilon}(\bar{r}(\rho), t)$, after taking into account that $\bar{r}(\rho)$ increases in $\rho$, $\epsilon(r)$ is non-decreasing in $r$ and $\bar{\epsilon}(r^t, t)$ decreases in $t$ and $r^t$. ■

47