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## UNDERINVOICING OF EXPORTS, OVERINVOICING OF IMPORTS, AND THE DOLLAR PREMIUM ON THE BLACK MARKET

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*Resumen:* Este artículo se divide en dos partes. La primera presenta un desarrollo teórico sobre el problema de sub—o sobre— facturación de exportaciones o importaciones en el contexto de un modelo de elección entre activos riesgosos y seguros. La segunda parte trata de aplicar este modelo a la economía brasileña. Se concluye que no puede ser rechazada la hipótesis de que la prima sobre el dólar en el mercado negro juega un papel en este proceso.

*Abstract:* This paper is divided into two parts. The first is a theoretical development of the question of under or overinvoicing of exports and imports within the context of a model of choice between risky and secure assets. The second part attempts to ascertain the empirical evidence for the Brazilian economy. It is concluded that the hypothesis that the black market dollar premium plays a role in this process can not be rejected.

### 1. Introduction

Within the international economic scenario of our days, one feature that is common to many countries, especially those that are in the process of development, is the existence of measures that restrict trade and the foreign

exchange market. As a result of this situation, black markets arise in which the foreign currency market enjoys a position of prominence. The existence of such markets, in which the exchange rate is almost always higher than the official exchange rate, gives rise to a premium in the sale and purchase of currency. This normally positive premium strongly induces manipulation of reports of international trade invoicing.

Exporters can boost their profits by underinvoicing and later on selling the currency corresponding to the underinvoiced amount on the black market, thereby obtaining a greater amount of local money for the same transaction. In the case of importers, the stimulus is in favour of overinvoicing the imports. Indeed, what happens is that when the importer overinvoices and sells on the black market the amount corresponding to the overinvoiced portion, at the close of the operation he manages to spend less local currency for the same import. It should be noted that, insofar as they are illegal practices, both over- and underinvoicing are subject to legal charges. These practices are therefore profitable when the black market-premium is greater than the risk premium in question.

This paper tries to ascertain the effect of the premium on the exchange-rate black market on this type of procedure. The paper is organized in the following way: the second section develops the question of under- or over-invoicing exports and imports within the context of the theory of choice involving risk by using an adaptation of a model (Arrow, 1974) of choice between risky and secure assets. In the third section are shown the results of an econometric model that seeks to investigate the empirical evidence for the Brazilian economy during the recent past with regard to this topic. In the fourth section are given the main conclusions reached by the paper.

## 2. The Theory of Choice Involving Risk and Under-Invoicing of Exports

The practice of an accounting report at dissonance with the true transaction is always a possibility whenever an economic agent is compelled by some mechanism to sell his endowments—or production—at a lower value than what another market could offer.

A typical case of this procedure is seen in the overinvoicing of imports and underinvoicing of exports. If  $F$  is the price at which the Central Bank buys or sells foreign money and  $F(1 + P)$ ,  $P > 0$ , the black market price, it obviously becomes advantageous, when other factors in the analysis are disregarded, to buy foreign money at  $F$  and sell at  $F(1 + P)$  (overinvoicing of imports), which is economically similar to not selling at  $F$  and selling at  $F(1 + P)$  (underinvoicing of exports).

What can offset this type of procedure are the risks involved when the Central Bank discovers fraud, and the consequences of this discovery. In attempting to formalize the decision-making mechanism on the part of some agent as regards his involvement in this type of practice, we shall use the following hypotheses:

a) The agent maximizes his expected utility from his income  $Y$ . This income derives from his exports carried out during a certain period of time and expressed in foreign currency.<sup>1</sup>

b) The agent must choose how much of this income ( $R = Y - B$ ) he will sell to the Central Bank at price  $F$  (including possible direct or indirect taxation). The remaining portion  $B$  will be sold on the black market at price  $F(1 + P) + \epsilon$ ,  $P$  (the black market premium) being given by  $(F_B - F)/F$ , with  $F_B$  translating the quotation in domestic currency of the foreign exchange rate quoted on the black market. The random variable  $\epsilon$  represents the random loss arising from the risks involved in the accounting fraud.

c) The black premium ( $P$ ) is constant and independent of the agent's decisions. This is equivalent to supposing that the agent is a "small" component of this market.

Given these hypotheses, our problem is reduced to finding the value of  $B$  in the interval  $0 \leq B \leq Y$ , which maximizes the expected utility from income:

$$G(B) = EU(YF + B(FP + \epsilon)) \quad (1)$$

For this, we differentiate expression (1) twice, and obtain:

$$G'(B) = E(FP + \epsilon)U'(YF + B(FP + \epsilon))$$

$$G''(B) = E(FP + \epsilon)^2U''(YF + B(FP + \epsilon))$$

If the agent is risk-lover,  $U''(\cdot)$  is by definition positive and function  $G(B)$  convex in  $B$ . In this case, the whole of income  $Y$  will be sold to the Central Bank if  $G(0) > G(Y)$ , or, on the black market when  $G(Y) > G(0)$ . If  $G(0) = G(Y)$ , the agent is indifferent as to one option or the other. In either case, no diversification is possible. The agent will sell all his income on one or the other market.

Given indifference to risk, we can take  $U(Y) = Y$ . Maximization of (1) will therefore have  $B = 0$  for a solution, if  $E\epsilon + FP < 0$ , or  $B = Y$ , if  $E\epsilon + FP > 0$ . In

<sup>1</sup> To simplify the exposition, we shall approach the problem here only from the point of view of underinvoicing of exports. Extending to the case of overinvoicing of imports follows a similar procedure.

the case where  $E\varepsilon + FP = 0$ , the agent is indifferent to any value of  $B$  in the interval  $[0, Y]$ .

It can be observed that under these two possible attitudes towards risk, there are no situations of a single optimal point that involves diversification, that is, compatible with a procedure by which part of the income in foreign currency is sold on the black market and another part sold to the Central Bank. This type of solution only occurs when the agent is supposed to be risk-averse. In this case, the function to be maximized ( $G(B)$ ) is concave and able to produce the three cases illustrated in Figure 1.

In Figure 1(a) the agent will sell nothing on the black market. One necessary and sufficient condition for this to occur is that

$$G'(0) = E(FP + \varepsilon)U'(FY) < 0,$$

or equivalently (since  $U'(RF) > 0$ ),  $E(FP + \varepsilon) < 0$ . This equation shows that the risk-averse individual will sell nothing on the black market if (and only if) expected amount of gain compared to the Central Bank (which is risk-free) is negative.

In Figure 1(b) all of the revenue in foreign currency is sold on the black market. This occurs when

$$G'(Y) = E(FP + \varepsilon)U'(Y(F(1+P) + \varepsilon)) > 0.$$

Finally, Figure 1(c) presents the most interesting case, in which the allocation of revenue is diversified, one part  $B^e$  being sold on the black market, and the remaining part ( $R^e = Y - B^e$ ) to the Central Bank. This takes place when  $E(FP + \varepsilon) > 0$  and  $G'(Y) < 0$ . The point of equilibrium  $B^e$  is thus determined by the equation:

$$H(B^e, FY, FP) = E(FP + \varepsilon)U'(FY + B^e(FP + \varepsilon)) = 0. \quad (2)$$

This equation can be solved, and we may write  $B^e$  as a function of  $FY$  and  $FP$ . That is:

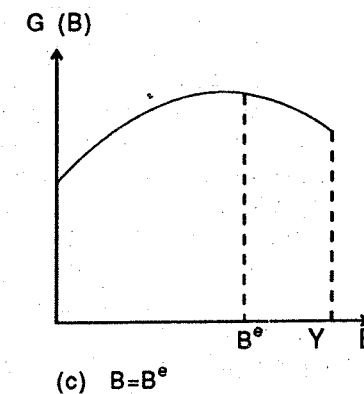
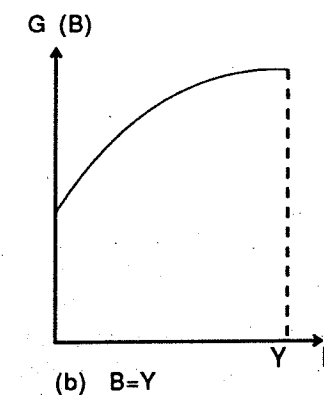
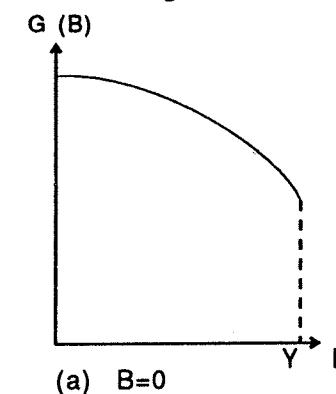
$$B^e = b(FY, FP) \quad (3)$$

or

$$R^e = Y - B^e = Y - b(FY, FP) = r(Y, FY, FP) \quad (4)$$

where  $R^e$  is the registered value of exports in Central Bank accounts.

Figure 1



We will show in the Appendix that if we suppose an absolute risk-aversion coefficient that decreases or is constant with income, it can be guaranteed that an increase in the exporter's total invoicing in domestic currency resulting from an increase in the initial invoicing in foreign currency, or from a devaluation of the exchange rate  $F$ , will lead to a rise in the underinvoiced total. In symbols:

$$\text{sgn} \frac{\partial B^e}{\partial FY} \cdot \text{sgn} A'(Y) < 0 \quad (5)$$

where  $A(Y) = -U''(Y)/U'(Y)$  is the coefficient of absolute risk-aversion.

Under this same hypothesis ( $A'(Y) \leq 0$ ), it can also be guaranteed (see Appendix) that a rise in the black market premium ( $P$ ) will lead to an increase in the underinvoiced total:

$$\frac{\partial B^e}{\partial FP} > 0 \quad (6)$$

In the case where  $A'(Y) > 0$ , this will occur if and only if (see Appendix):

$$EU'(YF + B^e(FP + \epsilon)) + B^e \frac{\partial H}{\partial FY} > 0 \quad (7)$$

Empirical observation of cases of underinvoicing of exports bears out several of the possibilities we have mentioned up to now. The simple act of smuggling is equivalent to a corner solution in which  $B = Y$ , in other words, all the exchange revenue obtained is destined for the black market. This would be compatible with any of the three attitudes towards risk: propensity (with  $G'(Y) > G'(0)$ ), indifference (if  $FP + E\epsilon > 0$ ), or aversion to risk (if  $G'(0) > 0$ ).

Total absence of any type of underinvoicing, which would be translated by  $B^e = 0$ , that is, by the sale of all exchange revenue to the Central Bank, also fits both into the propensity case (with  $G'(0) > G'(Y)$ ), the indifference case (if  $FP + E\epsilon < 0$ ), and the risk-aversion case (if  $G'(0) < 0$ ).

Intermediary behavior, however, by which underinvoicing means that a part of the total amount received in foreign currency is sold to the Central Bank and another part on the black market is only compatible (in the absence of other cost, as is supposed in this model) with the hypothesis of risk-aversion. In principle this is done by means of the difference between the price registered in the Central Bank accounts and what is actually received for each export unit. Furthermore, if underinvoicing is not an inferior good, that is, one that increases with the exporter's revenue, this is a sign that the latter demonstrates an absolute risk-aversion coefficient that decreases with income. In this case it can be guaranteed that a rise in the black market premium would lead to an increase in the portion of the total revenue from exports that is destined to supply to the black market.

### 3. Underinvoicing of Exports and Overinvoicing of Imports: Empirical Evidence

We next present two rather simple models on which we can base an analysis of the hypothesis that the premium on the black market dollar stimulates underinvoicing of exports and overinvoicing of imports.

In the case of exports, we admit that the export figure  $R$  officially registered in Central Bank is equal to the total value of export  $y$  minus the amount  $B$  of foreign currency sold in the black market. The export figure in equal to the dollar price index ( $\pi^*$ ) of Brazilian goods times the exported quantum ( $y$ ):

$$Y = \pi^* y \quad (8)$$

We assume that exported quantity  $y$  is a function of the effective real exchange rate  $e$ , the level of world income  $q$ , and the rate of utilization of the domestic productive capacity  $u$ . That is,

$$y = y(e, q, u) \quad (9)$$

By combining (4), (8) and (9) we obtain the following equation for the registered value of exports:

$$R = R(\pi^*, e, q, u, P) \quad (10)$$

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where the signs below each variable indicate the expected signs of the partial derivatives of this function.

The empirical research that is reported in this paper made use of quarterly data from the 1979 to 1985 period. The year of 1986 was disregarded, being an atypical year because of the well-known effects of the Cruzado Plano (see Barbosa, Faro and Brandão, 1989).

The 1979-85 period was characterized by the high rates fetched by the dollar premium on the black market, defined as the percentual difference between the exchange rate on the black market and the official market rate. Table 1 shows the evolution of the dollar premium during the period 1978-I-1985-IV. This period reveals not only high rates for the premium, but also a great variability, as can easily be seen by examining the values in Table 1.

Our study of exports concentrates on exports of industrialized products. The idea is that the practice of underinvoicing must be more common for this type of merchandise, which is characterized by a great heterogeneity of specifications and prices, than for agricultural products, which, besides being more homogeneous, mostly present prices that are quoted in international

commodity markets. The dollar values for exports of industrialized products are those published in the Bulletin issued by the Central Bank of Brazil.

**Table 1**  
The Dollar Premium on the Black Market

Year	Quarters				Annual Average
	I	II	III	IV	
1978	17.2	10.9	20.7	24.3	18.3
1979	16.3	20.0	14.6	23.1	18.5
1980	8.5	11.2	17.5	12.0	12.3
1981	2.3	11.7	24.1	27.4	16.4
1982	35.3	38.0	54.6	69.3	49.3
1983	69.7	52.5	69.3	43.3	58.7
1984	18.5	7.4	14.1	17.5	14.4
1985	15.3	18.1	31.8	34.5	24.9

From the practical point of view, the estimation of an equation with quarterly data poses some problems for dynamic specification, because theory does not usually indicate the lags involved in the different variables. On the other hand, the use of variables with lags generally brings problems of multicollinearity, which limits and complicates empirical investigation. Thus, we start from an unrestricted autoregressive distributed lag model (see Hendry, Pagan and Sargan, 1984) of equation (10). That is:

$$R_t = \alpha(L)R_{t-1} + \sum_{j=1}^5 \beta_j(L)Z_{jt} + \xi_t \quad (11)$$

where  $\alpha(L)$  is a polynomial in the lag operator  $L$  ( $LX_t = X_{t-1}$ ) of order  $m_0$ ,  $\beta_j(L)$  is a polynomial of order  $m_j$ ,  $\xi_t$  is assumed to be white noise and the variables  $Z_{jt}$  are given by:  $Z_{1t} = \pi_t^*$ ,  $Z_{2t} = e_t$ ,  $Z_{3t} = q_t$ ,  $Z_{4t} = u_t$ ,  $Z_{5t} = P_t$ . This model is usually represented by the symbol  $AD(m_0, m_1, \dots, m_5)$ .

The variable  $e_t$  is equal to the product of the dollar price of the exports, by the subsidy rate  $(1 + s)$  and by the nominal exchange rate, divided by the wholesale price index of Getulio Vargas Foundation. The price index for exports of the transformation industries, which is published in the *Conjuntura Econômica* review, was used as a proxy for the export price in dollars.

In the case of subsidies we made an approximation because of the lack of quarterly data for these times. As a starting point we used the indexes of annual subsidies calculated by Moreira and Baumann (1986), and adopted the hypothesis that the subsidy structure remains constant throughout the year.

For world income we adopted as a proxy the quantum index of world imports calculated from data published in *International Financial Statistics*.

In the case of the variable utilization of installed capacity, we opted for the index for utilization of installed capacity calculated by Getulio Vargas Foundation and published in *Conjuntura Econômica* review.

Table 2 contains the estimated parameters of a restricted specification of equation (11), all the variables being in natural logarithms, except for the intercept and the seasonal dummies for the first and second quarters. The third quarter dummy was not significant, and thus it was removed the regression.

**Table 2**  
The Dollar Premium and Underinvoicing of Exports

Constant	Dummy 1	Dummy 2	$\pi_t^*$	$e_t$	$q_{t-2}$	$u_t$	$P_{t-2}$
1.584 (0.506)	-0.074 (-1.370)	-0.142 (-2.552)	0.095 (0.305)	0.893 (2.574)	2.763 (6.341)	-3.022 (-5.708)	-0.088 (-2.610)
$R^2 = 0.85$	D.W. = 1.57	LA (4, 14) = 0.74	HET (1, 24) = 0.01	$\hat{\sigma} = 0.10$			
$\bar{R}^2 = 0.79$	F (7, 18) = 14.46	NORM [ $\chi^2_{(2)}$ ] = 0.65	RESET (1, 17) = 0.35	F (10, 8) = 0.88			

Notes: 1) Figures between parentheses are student  $t$  statistics. 2) Method used: ordinary least squares.

With regard to the variables in (10), all the variables have the expected signs and, except for the variable for price of exports in dollars, are statistically significant at the 5% level.

The coefficient of the price index for exports in dollars is not significantly different from zero from the statistical point of view. This result led us to look closely at the behavior of this index during the period analysed. The variability of the index is quite small and its value relatively stable. Consequently, the variation of the dollar value of exports in this period is due primarily to the variation of the exported quantum. It is not surprising, therefore, that the price index in dollars did not affect the dollar value of the exports.

The conclusion reached by various pieces of research in Brazil as to the main variables that affect exports of manufactured goods is repeated here,

since the real exchange rate, world income, and level of utilization of capacity are all significant and of similar magnitude to other studies already carried out on the subject of Brazil's economy.

The dollar premium on the black market has a negative effect on the dollar value of exports and is significantly different from zero. This result is consistent with the hypothesis of overinvoicing of exports.

The estimates given in Table 2 resulted from a restricted version of (11). The lags with respect to world income and the dollar premium on the black market are of two quarters, while the other variables have no delay. The lagged dependent variable was included, but it was removed based on an  $F$  test.

Table 2 also reports the values obtained for several specification tests. LA (4, 14) is an  $F$ -version of Lagrange multiplier test for 4th order residual outorrelation; NORM is a test for normality of residuals based on skewness and kurtosis and follows a chi-square distribution with two degrees of freedom; HET(1, 24) is an  $F$ -version of Lagrange multiplier test for heteroscedasticity; and RESET(1, 12) is an  $F$ -version of Ramsey's (1969) test for functional form misspecification.<sup>2</sup> None of the test statistics indicate any serious mis-specification of the equation reported in Table 2.

In the case of imports, we assume that the import figure  $R$  officially registered in Central Bank is equal to the total value of imports plus the amount  $B$  of foreign currency sold in the black market. The import figure is equal to the dollar price index of foreign goods ( $\pi$ ) times the imported quantum ( $m$ ):

$$y = \pi m \quad (12)$$

In the imports model we use the traditional equation of imports, in which the imported quantum is a function of the real cost of imports ( $e^*$ ) and of the real income level of the economy ( $q^*$ ). That is:

$$m = m(e^*, q^*) \quad (13)$$

The quantity imported and the real income of the economy vary in the same direction ( $\partial m / \partial q^* > 0$ ), whereas the real cost of imports and the imported quantum must be negatively correlated ( $\partial m / \partial e^* < 0$ ).

By combining equations (12), (13) and (3), the following equation is reached for the registered value of imports in dollars:

$$R = R(\pi, e^*, q^*, p) \quad (14)$$

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where, once again, the signs below each variable indicate the expected signs of the respective partial derivatives.

The import data used for the empirical analysis, with all the variables in their natural logarithms, were total imports except for oil and wheat. The exclusion of these two products is reasonable, given the specific characteristics that define the process of their importation by the government.

As for the real exchange rate  $e_t^*$ , we adopted the hypothesis that the tariff structure did not alter significantly during the period. The real exchange rate is therefore equal to the product of the nominal exchange rate by the import price index in dollars (except for oil and wheat), divided by the wholesale price index of Getulio Vargas Foundation. The dollar price index for imports was obtained directly from the Getulio Vargas Foundation.

The domestic income variable is given by the gross domestic product (GDP) of the country. Although the publication of statistics related to the GDP in quarterly terms is common in developed countries, it is not practised by official organs in Brazil. We therefore resort to a paper by Lopes and Modiano (1985) which includes estimates for the quarterly GDP.

As in the case of exports, we start from an unrestricted

$$AD(m_0, m_1, \dots, m_4)$$

model:

$$R_t = \gamma(L)R_{t-1} + \sum_{j=1}^4 \delta_j(L) x_{jt} + \eta_t \quad (15)$$

where  $\gamma(L)$  is a polynomial in the lag operator  $L$  of order  $m_0$ ,  $\delta_j(L)$  is a polynomial of order  $m_j$ ,  $\eta_t$  is a white noise random variable, and the variables  $x_{jt}$  are given by:  $x_{1t} = \pi_t$ ,  $x_{2t} = e_t^*$ ,  $x_{3t} = q_t^*$  and  $x_{4t} = P_t$ .

Table 3 presents the results obtained in estimating the parameters of a restricted version of equation (15) by the method of ordinary least squares. The coefficients of the real exchange rate, real income, and price index in dollars all show the expected signs and are significant from the statistical point of view. The dummy variable for the second quarter was significant, whereas the others dummies were insignificant. The registered value of imports with four lags also entered as an explicative variable.

The estimate of the coefficient with two lags of the dollar premium on the black market has a negative sign, whereas the coefficient for three lags has a positive sign. The sum of these two coefficients is positive (0.007) and its standard error is 0.026. Therefore, the hypothesis that the sum of the

<sup>2</sup> Krämer and Sannberger (1986) gives an exposition of several of these mis-specification tests

dollar premium on the black market stimulated overinvoicing of imports would be rejected. Table 3 presents also the results of several specification tests. Again, none of the statistics point out mis-specification of the estimated equation.

**Table 3**  
The Dollar Premium and Overinvoicing of Imports

Constant	Dummy 2	$R_{t-4}$	$\pi_t$	$\pi_{t-1}$	$\pi_{t-3}$	$\pi_{t-4}$	$\epsilon_t^*$	$q_{t-1}^*$	$P_{(-2)}$	$P_{(-3)}$
-21.086 (-7.017)	0.215 (4.469)	0.384 (4.942)	1.625 (3.887)	0.848 (2.517)	0.939 (1.417)	-0.521 (-2.220)	-0.939 (-2.886)	2.963 (6.204)	-0.065 (-2.307)	0.072 (2.467)
$R^2=0.97$	D.W. = 1.90	LA (4, 9) = 2.81		HET(1,22) = 0.07		$\hat{\sigma} = 0.07$				
$\bar{R}^2=0.94$	F(10,13) = 37.19	NORM [ $\chi^2_{(2)}$ ] = 0.89		RESET(1,12) = 8.45		F(9,4) = 2.23				

Notes: 1) Figures between parentheses are student *t* statistics. 2) Method used: ordinary least squares.

#### 4. Conclusions

This paper has presented a theoretical model as well as an empirical analysis of the question of under- or over- invoicing of exports and imports, where the black market premium on the foreign exchange rate is supposed to play a role in this process. It reached the following conclusions:

i) Empirical evidence for the period 1979-IV-1985-IV in exports of Brazilian manufactured goods is consistent with the hypothesis that the dollar premium on the black market stimulated the practice of underinvoicing exports;

ii) The hypothesis that the premium on the black market caused overinvoicing of total Brazilian imports, excluding petrol and wheat, is rejected for the period 1979-IV-1985-IV;

iii) The existence of a registered value of exports sensitive with respect to the black premium renders the empirical results compatible with the theoretical developments carried out in the second section of the paper. If we suppose risk aversion and risk-aversion coefficient that decreases with income, this implies an increase in underinvoicing of exports when the premium on the black market rises. This being the behavior of the economic agents, it can also be claimed that when the total initial revenue in local currency derived from exports is increased, whether due to a devaluation of the exchange rate or to an increase in revenue from exports, there will also be an increase in the amount destined to be sold on the black market.

#### Appendix

PROPOSITION 1. Let  $\frac{\partial B^e}{\partial FY}$  and  $\frac{\partial B^e}{\partial FP}$  be the partial derivatives of (3) with respect to  $FY$  and  $FP$ ;  $\frac{\partial H}{\partial FY}$  and  $\frac{\partial H}{\partial FP}$  be the partial derivatives of (2) with respect to  $FY$  and  $FP$ . Then,

$$\text{sgn} \frac{\partial B^e}{\partial FY} = \text{sgn} \frac{\partial H}{\partial FY}$$

$$\text{sgn} \frac{\partial B^e}{\partial FP} = \text{sgn} \frac{\partial H}{\partial FP}$$

PROOF. To begin, let us note that from equation (2)

$$\frac{\partial H}{\partial B^e} \frac{\partial B^e}{\partial FY} + \frac{\partial H}{\partial FY} = 0$$

$$\frac{\partial H}{\partial B^e} \frac{\partial B^e}{\partial FP} + \frac{\partial H}{\partial FP} = 0$$

With  $\partial H / \partial B^e = G''(B^e) < 0$ , it is concluded that  $\partial B^e / \partial FY$  will have the same sign as  $\partial H / \partial FY$ , the same happening with  $\partial B^e / \partial FP$  and  $\partial H / \partial FP$ . ■

PROPOSITION 2. If  $\frac{\partial H}{\partial FY} > 0$  then  $\frac{\partial H}{\partial FP} > 0$ . On the other hand if  $\frac{\partial H}{\partial FY} < 0$  we have  $\frac{\partial H}{\partial FP} \leq 0$ .

PROOF. From equation (2):

$$\frac{\partial H}{\partial FY} = E(FP + \epsilon) U''(YF + B^e(FP + \epsilon)) \quad (\text{A.1})$$

$$\frac{\partial H}{\partial FP} = EU''(YF + B^e(FP + \epsilon)) + B^e E(FP + \epsilon) U''(YF + B^e(FP + \epsilon)) \quad (\text{A.2})$$

or again, from (A.1) and (A.2):

$$\frac{\partial H}{\partial FP} = EU'(YF + B^e(FP + \epsilon)) + B^e \frac{\partial H}{\partial FY} \quad (\text{A.3})$$

As  $U'(YF + B^c(FP + \epsilon))$  is necessarily positive, since it represents the marginal utility of income, the sign of  $\frac{\partial H}{\partial FP}$  will be positive if  $\frac{\partial H}{\partial FY} > 0$ , and may be positive, negative or null if  $\frac{\partial H}{\partial FY} < 0$ . ■

PROPOSITION 3. The partial derivative  $\frac{\partial H}{\partial FY}$  will have the opposite sign to that of  $A'(Y)$ :

$$\text{sgn } \frac{\partial H}{\partial FY} \cdot \text{sgn } A'(Y) < 0 \quad (\text{A.4})$$

PROOF. Let us start by taking  $A'(Y) < 0$ . In this case, for any value of  $(FP + \epsilon)$ ,

$$(FP + \epsilon)A(YF + B^c(FP + \epsilon)) < (FP + \epsilon)A(YF)$$

By multiplying both terms by the negative value of  $U'(YF + B^c(FP + \epsilon))$  we get:

$$(FP + \epsilon)U''(YF + B^c(FP + \epsilon)) > -A(YF)(FP + \epsilon)U'(YF + B^c(FP + \epsilon))$$

Taking the mathematical expectation,

$$\begin{aligned} \frac{\partial H}{\partial FY} &= E(FP + \epsilon)U''(YF + B^c(FP + \epsilon)) \\ &> -A(YF)E(FP + \epsilon)U'(YF + B^c(FP + \epsilon)) \end{aligned}$$

But from equation (2), the second member of the above equation is equal to zero, which gives us:

$$\frac{\partial H}{\partial FY} > 0$$

A similar argument could be used to show that  $\frac{\partial H}{\partial FY} < 0$  if  $A'(Y) < 0$ , and  $\frac{\partial H}{\partial FY} = 0$  if  $A'(Y) = 0$ . ■

PROPOSITION 4. If the absolute risk-aversion coefficient decreases or is constant with income, then an increase (decrease) in the exporter's total invoicing in domestic currency resulting from an increase (decrease) in the initial invoicing in foreign currency, of from a devaluation of the exchange rate  $F$ , will lead to a rise (decline) in the undervoiced total. ■

PROPOSITION 5. An increase (decrease) in the black market premium will always lead to an increase (decrease) in the total invoiced amount if the absolute risk-aversion coefficient is a constant or decreasing function with respect to income. In the case where  $A'(Y) > 0$ , this will occur if and only if (7) holds. ■

PROOF. By combining Propositions 1, 2 and 3, it is easy to see that if  $A'(Y) > 0$ , then  $\frac{\partial B^c}{\partial FP} > 0$ . By Proposition 3 when  $A'(Y) > 0$ ,  $\frac{\partial H}{\partial FY} < 0$ . Therefore  $\frac{\partial H}{\partial FP} > 0$  if and only if (7) holds, as can be seen by looking at (A.3). ■

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